PHYSICS 211

NEWEST QUIZZES & FINALS

1997 QUIZZES ARE INCLUDED
1998 – 1999 EDITION

AMERICAN UNIVERSITY OF BEIRUT

Physics 211

Quiz I

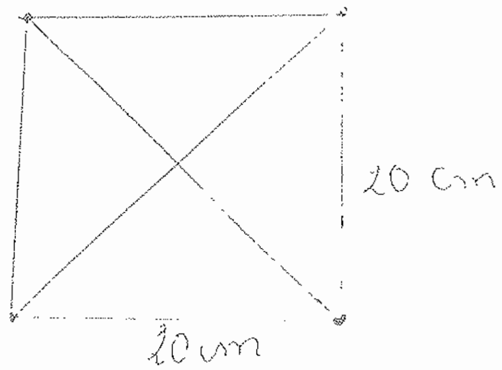
Time : 1 hour April 8, 1998

Question I 15 %

Four identical point charges of Q coulomb are located at the corners of a square.

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \, Nm^2 / C^2$$

$$Q = 5 \times 10^{-6} C$$



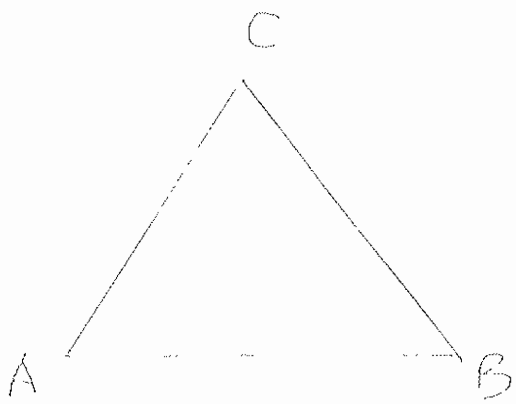
a) How much work must be done to completely separate the four charges from one another ?

b) Find the force (magnitude and direction) ? one point charge of 2Q coulomb located at the center of the square .

c) Find the force (magnitude and direction) on the charge at the center of the square when one of the corner charges is removed.

Question II 15 %

The electric potential at the A, B & C, the corners of an equilateral triangle with a side of 0.5 m, are respectively : +10V, +15V, & -12V. How much work (sign & magnitude in units of eV, the charge of the proton is +e) must be done to move a proton at constant speed.



a) From A via B to C
b) From A via C to B
c) From A via C and B to A
d) From C to A

Solution :

Question I

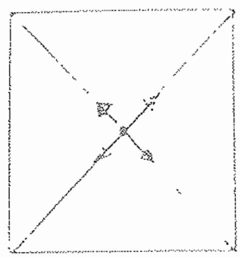
a) Energy stored in the system of four charges :

$$u = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q^2}{a} + \frac{Q^2}{a} + \frac{Q^2}{a\sqrt{2}} + \frac{Q^2}{a} + \frac{Q^2}{a} + \frac{Q^2}{a\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q^2}{a}\right][4 + \sqrt{2}] = 6.1J$$

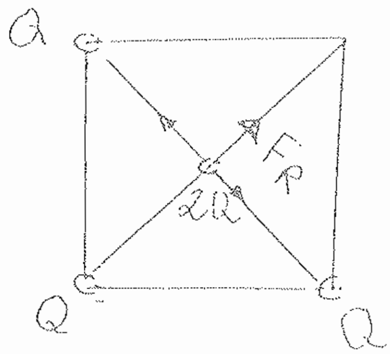
To separate charge : -6.1 J

b) $\sum F = 0$



c) $F_R = \dfrac{Q \times 2Q}{4\pi\varepsilon_0 \times (\frac{1}{2} a\sqrt{2})^2} = 22.5N$

direction to the empty corner



Question II

a) $\equiv$ A $\rightarrow$ C

$V_C - V_A = \Delta V = -12 - 10 = -22$ V

$W = q\Delta V = -22$ eV

b) $\equiv$ A $\rightarrow$ B

$V_B - V_A = \Delta V = 15 - 10 = 5$ V

$W = q\Delta V = 5$ eV

c) $\equiv$ A $\rightarrow$ A

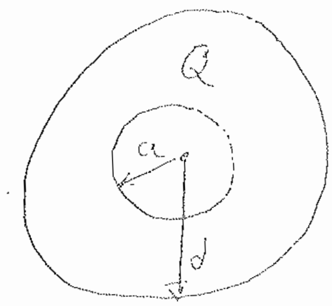
$\Delta V = 0$ V

$W = 0$ V

d) $\equiv$ $-(A \rightarrow C)$

$W = 22$ eV

Question III



a) $Q1 = -Q$

b)

$$V = -\int_{\infty}^{r=b} \vec{E}.d\vec{s} = -\int_{\infty}^{r=b} \frac{q_{net}}{4\pi\varepsilon_0 r^2} dr = \frac{q_{net}}{4\pi\varepsilon_0 b}$$

$q_{net} = Q + Q1 + Q2$ ---------> $Q2 = q_{net} = 4\pi\varepsilon_0 b V$

c)

$r > b$:

$$E = \frac{q_{net}}{4\pi\varepsilon_0 r^2} = \frac{4\pi\varepsilon_0 bV}{4\pi\varepsilon_0 r^2} = \frac{bV}{r^2}$$

$a < r < b$:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$r \lesssim a$:

$$E = 0$$

Question IV 30%

Two large, parallel, metal plates of area 4 m^2 face each other & carry equal and opposite charges on their inner surfaces. The potential difference between the plates is 550 V. A proton, placed at a distance of 3 cm from the positively charged plate experiences a force of 8.8×10^{-16} N

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \, Nm^2 / C^2$$

$$e = 1.60 \times 10^{-19} \, C$$

$$m_p = 1.67 \times 10^{-27} \, Kg$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \, F / m$$

a) Find the distance d between the plates ?

b) What is the total charge on the plates ?

c) Another proton, placed midway between the two plates, moves towards the negatively charged plate with a velocity V_0, what should the magnitude & direction of V_0 be in order that the proton just comes at rest in front of the negatively charged plate ?

Question III 40 %

A spherical conductor of radius a has a charge Q placed on it. This spherical conductor is surrounded by a thin, conducting spherical shell (concentric) of radius b. The shell is kept at constant potential V.

a) Find the charge Q1 on the inner surface of the shell ?

b) Find the charge Q2 on the outer surface of the shell ?

c) Find the electric field at a distance r from the center of the spherical conductor where $r > b$; $a<r<b$; & $r < a$. ?

d) Find the electric potential at a distance $r > b$; $a<r<b$; & $r < a$. ?

d)

$r > b$:

$$V = -\int_{\infty}^{r=b} \vec{E}.ds = \int_{\infty}^{r} Eds = -\int_{\infty}^{r} Edr = -bV\int_{\infty}^{r} \frac{dr}{r^2} = +bV[1/r]_{\infty}^{r} = \frac{bV}{r}$$

$a < r < +b$:

$$V = -\int_{\infty}^{r=b} \vec{E}.ds - \int_{b}^{r} \frac{Q}{4\pi\varepsilon_0 r^2} dr = V + \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 b}$$

$r < a$:

$$V_p = V + \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

<u>Question IV</u>

a)

$F = eE = 8.8 \times 10^{-16}$ N
$E = (8.8 \times 10^{-16}$ N$)/(1.60 \times 10^{-19}$C$) = 5500$ N/C
$V = Ed = 550$ V -----> $d = 550/E = 0.1$m

b)

$$E = \frac{\sigma}{\varepsilon_0} \quad \rightarrow \quad \sigma = 8.85 \times 10^{-12} \times 5500 = 4.9 \times 10^{-8}\ C/m^2$$

$$Q = Area \times \sigma = 4 \times 4.9 \times 10^{-8} = 19.6 \times 10^{-8}\ C$$

c)

$$\Delta V = 275V$$

$$\Delta U = e\Delta V$$

$$\frac{1}{2} mV_0^2 = e\Delta V$$

$$V_0^2 = \frac{2e\Delta V}{m} = 527 \times 10^8$$

$$V_0 = 2.3 \times 10^5\ m/s$$

towards the +vely charged plate.

AMERICAN UNIVERSITY OF BEIRUT

Physics 211

Quiz III

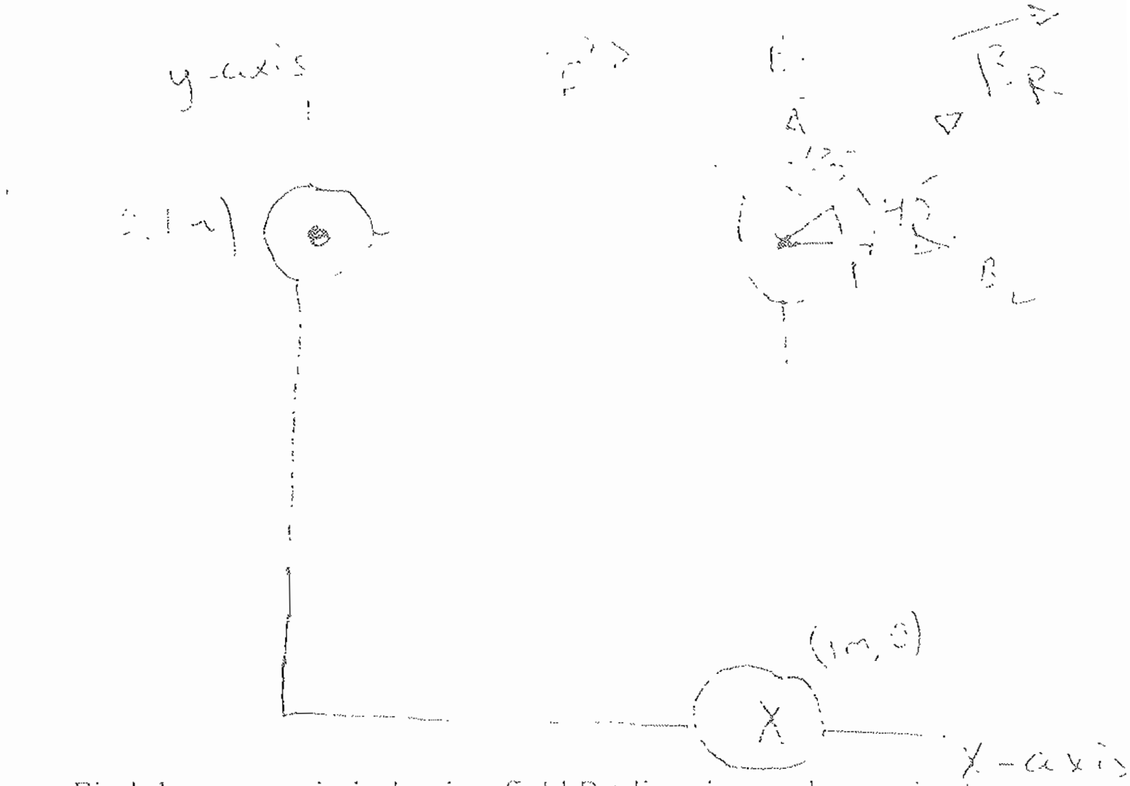
Time : 1 hour June 10, 1998

Name --------------------

I.D. No. ------------------

Question I. (20%)

Two long parallel wires run perpendicular to the xy plane. Wire 1 intersects the xy plane at the y-axis at the point (0, 1m). Wire 2 intersects the xy plane on the x-axis at the point (1m, 0). Wire 1 carries a current of 100 A out of the xy plane, and wire 2 carries a current of 100 A into the xy plane. ($\mu_0 = 4\pi \times 10^{-7}$ H/m)



a) Find the magnetic induction field B (direction and magnitude) at a point p in the xy plane, located at (1m, 1m).

b) A third long wire is parallel to the 2 other wires & passes through the point p. If the current in this third wire is 1A, directed out of the xy plane, find the magnitude and direction of the force per unit length exerted on the wire.

Question II. (30%)

A very long solenoid is wound with 20 turns of wire per cm and the current in its winding is increasing at the rate of 200 A/s. The diameter of the solenoid is 20 cm. A point charge of 5C is placed at a distance of 5 cm from the axis of the solenoid, away from the ends of the solenoid.
($\mu_0 = 4\pi \times 10^{-7}$ H/m)

What is the force, if any, acting on the point charge ?

Question III. (20%)

A coil of wire has a radius of 10.0 cm, 100 turns and a total resistance of 100 ohms.

a] At what rate must a magnetic induction field B, perpendicular to the plane of the coil, change to produce Joule heating in the coil at the rate of 10 mW ?

b] If the magnetic induction field B is decreasing, what direction should the magnetic field have with repect to the plane of the coil in order to produce a counter-clockwise current ? Make the drawing.

Question IV. (30%)

A very long empty solenoid has 50 turns per cm and carries a currewnt of 10A. ($\mu_0 = 4\pi \times 10^{-7}$ H/m)

a) What is the magnetic field inside the empty solenoid on its axis & 3 cm away from its axis ??

b) The solenoid is then completely filled with a material of permeability $k_m = 250$. What is now the magnetic field inside the solenoid.

c) What is the magnetization M induced in the material by the applied field ?

Detailed Solution

Question I.

a)

$$B_1 = \frac{\mu_0}{2\pi} \times 100 \qquad B_2 = \frac{\mu_0}{2\pi} \times 100$$

$$B_R = \frac{\mu_0}{2\pi} \times [10^4 + 10^4]^{1/2} = 2.8 \times 10^{-5}\,T$$

b)

$$\vec{F} = i\vec{l} \times \vec{B}_R$$

$$F / l = iB_R = 1x2.8x10^{-5} = 2.8x10^{-5}$$

135° with the positive x - axis

Question II.

$$\phi_B = BA = \mu_0 niA$$

$$\frac{d\phi_B}{dt} = \mu_0 nA \frac{di}{dt} = 4\pi x10^{-7} x2x10^3 x\pi x(10^{-1})^2 x200 = 1.58x10^{-2}$$

$$\oint \vec{E}.d\vec{s} = -\frac{d\phi_B}{dt} \times \frac{\pi x(0.05)^2}{\pi x(10^{-1})^2}$$

$$E2\pi r = -1.58x10^{-2} x(1/4)$$

$$(E) = \frac{1.58x10^{-2}}{2\pi x0.05} = 0.05V/m \times (1/4)$$

$$F = qE = 5 \times 0.05 = 0.25 \times (1/4) = 0.0625N$$

Question III.

a)

$r = 10cm$

$N = 100$

$R = 100$ ohms

$$\varepsilon = -\frac{d\phi_B}{dt} N = -N\pi r^2 \frac{dB}{dt}$$

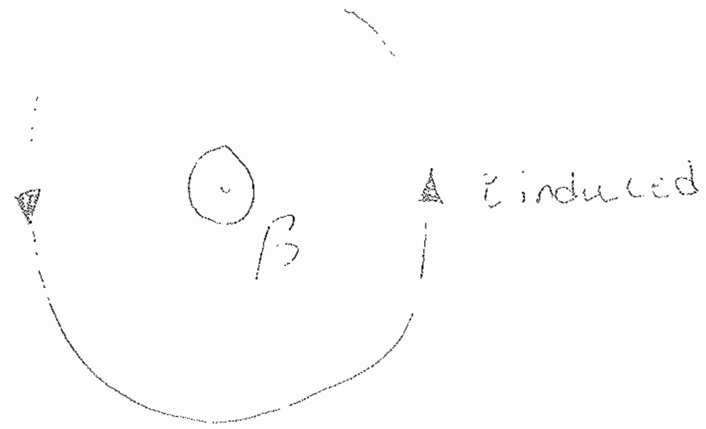
$$i = \frac{\varepsilon}{R} = \frac{-N\pi r^2}{R} \frac{dB}{dt}$$

$$\text{Joule Heating : } P = i^2 R = \frac{N^2\pi^2 r^4}{R}\left(\frac{dB}{dt}\right)^2 = 0.01$$

$$\frac{dB}{dt} = \left(\frac{10^{-2} R}{N^2\pi^2 r^4}\right)^{1/2} = \left[\frac{10^{-2} \times 100}{10^4 \times \pi^2 \times 10^{-4}}\right]^{1/2} = \frac{1}{\pi} = 0.32 T/s$$

b)

$$\frac{dB}{dt} < 0$$



Question IV.

a)

axis : $B_0 = \mu_0 in = 4\pi \times 10^{-7} \times 10 \times 5 \times 10^3 = 0.06T$

3 cm away from the axis :

the same 0.06 T

uniform field if inside

$B = 0$ if outside

b)

$B = k_m B_0 = 250 \times 0.06 = 15T$

c)

$B = B_0 + \mu_0 M$

$$M = \frac{B - B_0}{\mu_0} = \frac{(k_m - 1)B_0}{\mu_0} = \frac{249 \times 0.06}{4\pi \times 10^{-7}}$$

$$= 1.2 \times 10^7 \, Tm/H = 1.2 \times 10^7 \, A/m$$

<u>Physics 211</u> <u>Final Exam</u> Feb. 6, 1997

Name:

I.D. #

Section or Name of Prof.:

All problems are worth the same % of the total mark.

<u>General Information:</u>

Vacuum electric permitivity $\varepsilon_o = 8.85 \times 10^{-12}$ S I

Vacuum magnetic permitivity $\mu_o = 4\pi \times 10^{-7}$ S I

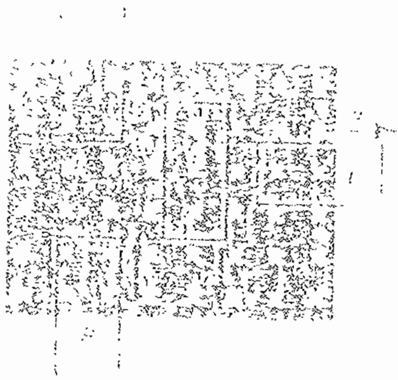
Electronic charge: $e = 1.60 \times 10^{-19}$ C

1. A permanently magnetized sphere of radius 25 mm has a uniform magnetization of magnitude 8400 (SI Unit) ______

(a) Indicate above the SI Unit of magnetization.
(b) Determine the magnitude of the magnetic moment of the sphere.
(c) Sketch the lines representing B inside and outside the sphere. Assume that B is uniform inside the sphere.

2. Consider an LC circuit with $L = 5.3$ mH, $C = 17$ nF, the initial charge of the capacitor is 2.2 μC, and the initial current in the circuit is zero. Write expressions for q, i, U, U_E, U_B as functions of t.

[illegible]



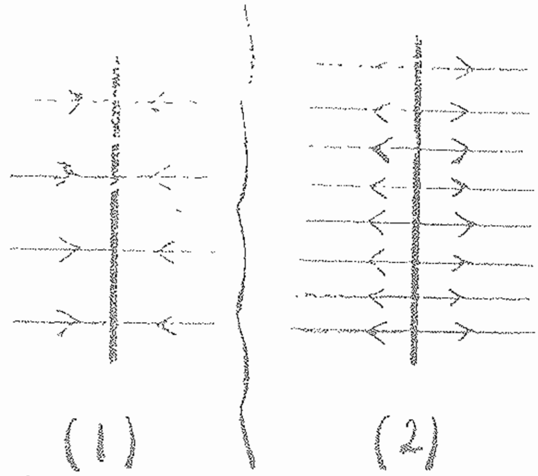
[illegible] capacitors [illegible] capacitance [illegible] figure [illegible] the greatest energy? [illegible]

[illegible] capacitor is charged by a [illegible] V dc power supply. The fully charged capacitor is then [illegible] through a 1.0 MΩ resistor. What are the initial rates of decrease of (a) the capacitor charge [illegible] capacitor energy?

The figures below show the electric field lines of force for two large sheets (1) and (2) (viewed sideways) carrying uniform surface charge density σ_1 and σ_2.

a) Find the ratio σ_1/σ_2.

b) What are the signs of σ_1 and σ_2?



A conducting sphere of radius 10 cm has a potential of [illegible]. Find (a) the surface charge density; (b) the electric field just outside the surface.

12. True or False

___ The conductivity of a metal generally increases with increasing temperature because thermal agitations enhance the electrons' drift velocity.

___ A charged particle moves through a constant magnetic field. The Lorentz force on this particle increases its kinetic energy.

___ The net force on a N-turn circular coil carrying a current I in a uniform magnetic field B that is directed perpendicular to the plane of the coil is proportional to the product INB.

___ Magnetic field lines start at the North poles and end at the South poles.

___ In diamagnetic materials the magnetization **M** is always opposite the external field **B**.

___ The potential energy of an electric dipole is minimum when it is aligned with a uniform electric field and maximum when it is perpendicular to the field.

___ The electric field and potential are zero inside an isolated conductor carrying an excess positive charge.

___ The charge on an isolated charged capacitor increases when a dielectric fills the capacitor.

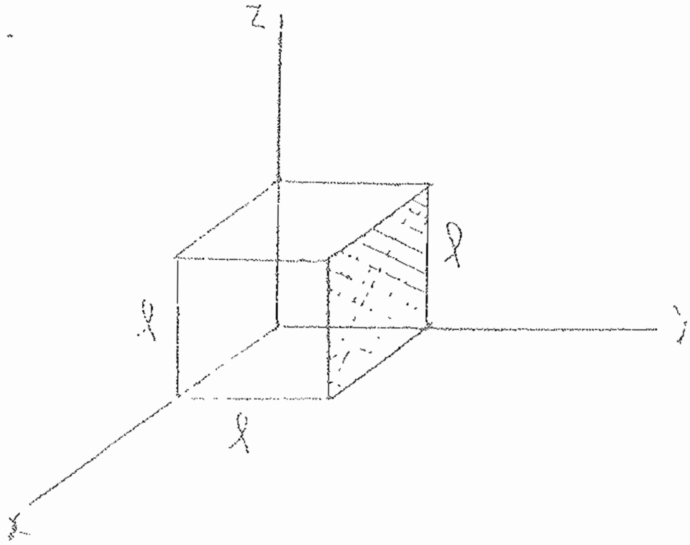
___ The Hall effect enables us to find the sign of the charge carriers.

___ For all conductors, ohmic and non-ohmic, resistance can be obtained if the voltage versus current dependence is known.

13. A cube of edge length $l = 2.5$ cm is positioned as shown in the figure below. There is a uniform magnetic field throughout the region given by: $\mathbf{B} = (5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ tesla.

a) Calculate the flux through the shaded face of the cube.

b) Find the total flux through the six faces of the cube.



14. The current I in a conductor depends on time as $I =$ [illegible] $+ 7$, where t is in seconds. What quantity of charge moves across a section through the conductor during the interval $t =$ [illegible] to $t =$ [illegible]

15. An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10 μF capacitor, the voltage across the combination becomes 30 V. Calculate the unknown capacitance in μF.

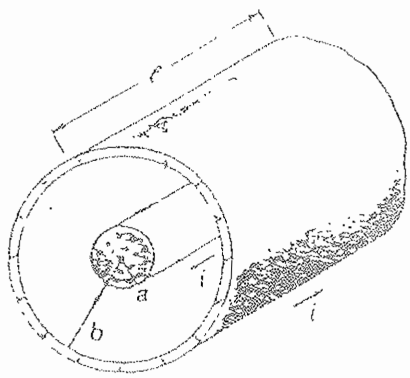
16. A 30 turn circular coil of radius 4 cm and resistance 1Ω is placed in a magnetic field directed perpendicular to the plane of the coil. The magnitude of the magnetic field varies in time according to the expression: $B = 0.01t + 0.04t^2$, where t is in second and B is in tesla. Calculate the current that can be observed in the coil at $t = 5$ s

17. A long coaxial cable with a circular cross section is shown in the figure below. The inner and outer conductors, having radii a anb b respectively, carry current i with opposite senses.

(a) Determine the energy density at a point between conductors.

(b) Determine the energy stored in the space between conductors for a length l of the cable.

(c) Estimate the self-inductance per unit length of this coaxial cahle. Consider only the empty region between a and b.



A.U.B.	Physics 211	Feb. 6, 1996
Physics Department	FINAL EXAM	Time 2 hours

Don't forget to write your name, your I.D.# and your section on your booklet.

Part A 30 min.

20 marks

A1) A small water droplet has a diameter of 4 μm. If it carries a charge of $12\ e^-$.

a)-what is the strength and direction of the uniform electric field that will just balance the gravitational force acting on the droplet?

b)-If the magnitude of the electric field is then increased to three times its previous value, how will the droplet behave compared with its behavior in the total absence of the electric field? Given: $e = 1.6\times10^{-19}$ C, the density of water is $1 g/cm^3$. Take $g=10\ m/sec^2$.

15 marks

A2) A current of 2.00 A flows in a copper wire of $1.00\ mm^2$ cross section. How long does it take an electron to travel 10 cm in this wire under these circumstances ? (Assume that each Cu atom contributes one conduction electron). $\rho_{Cu} = 8.92\ g/cm^3$, $M^{Cu}_{mole} = 63.5$ g/mol. and $N_A = 6.02\times10^{23}$.

15 marks

A3) A toroidal coil is wound with 4000 turns of wire. The average radius of the toroid is 10 cm and the diameter of the coils is 1.5 cm. A second coil of 400 turns is wound over the first.

a)- What emf is induced in the second coil if the current in the 4000-turns coil is changed at a rate of 25 A/sec?

b)- Discuss also the sense of this induced emf.

Part B 30 min.

15 marks

B1)- An electron half-way between two fixed protons on the x-axis, is in stable equilibrium and executes simple harmonic motion after a slight displacement along the y-axis. Find the restoring force on the electron and deduce its natural frequency in terms of its mass m and charge e and proton-proton separation d.

15 marks

B2)- A cylindrical capacitor has radii $a < b$, length L, and carries a charge q.

a)- Find an expression for its capacitance.

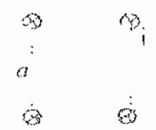
b)-Calculate the electrical energy stored if $a = 4.4$ mm, $b = 5.0$ mm, $L = 5.0$ cm, $q = 6.0\ \mu C$ and $(4\pi\varepsilon_o)^{-1} = 9\times10^{9}\ N.m^2/C^2$.

20 marks

B3)- Four long parallel wires forming the corners of a square, side a, run perpendicular to the page and carry equal currents i each into page.

a)- Find the magnetic field $\vec{B}$ at one corner due to the currents at the other corners.

b)-Calculate the magnetic force per meter exerted by $\vec{B}$ on the wire of that corner if $i = 2.0$ A, $a = 50$ cm, $\mu_o = 4\pi\times10^{-7}$ T.m/A .



Part C 30 min.

15 marks

C1)- Two identical raindrops, each carrying surplus electrons on its surface to make a net charge -q on each, collide and form a single drop of larger size. Before the collision, the characteristics of each drop are the following : a) surface charge density σ_o, b) electric field $\vec{E}_o$ at the surface, c) electric potential V_o at the surface (where $V = 0$ at $r = \infty$). For the combined drop, find these three quantities in terms of their original values. (Hint: use the fact that the volume is conserved).

20 marks

C2)- An α particle ($q = +2e$, $m = 4.003$ u) travels in a circular path of radius 4.5 cm in a magnetic field with $B = 1.2T$. Calculate a) its speed, b) its period of revolution, c) its kinetic energy in eV, and d) the potential difference through which it would have to be accelerated to achieve this energy. ($u = 1.661\times10^{-24}$ g , $e = 1.60\times10^{-19}$ C) .

15 marks

C3)- At $t = 0$, a source of emf, $\mathcal{E} = 500$ V, is applied to a coil that has an inductance of 0.80 H and a resistance of 30 Ω.

a)- Find the energy stored in the magnetic field when the current reaches half its maximum value.

b)-How long after the emf is connected does it take for the current to reach this value?

Part D 30 min.

16 marks

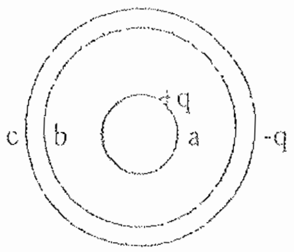
D1)- A uniform conducting sphere of radius (a) carries a charge (+q). It is placed at the center of a spherical conducting shell of inner radius (b) and outer radius (c). The outer shell carries a charge of (-q). Find:

a)- E(r) within the sphere ($r<a$).

b)- E(r) between the sphere and the shell. ($a<r<b$).

c)-E(r) outside the shell ($r>c$).

d)- Determine the charges which will appear on the inner and outer surfaces of the shell.



18 marks

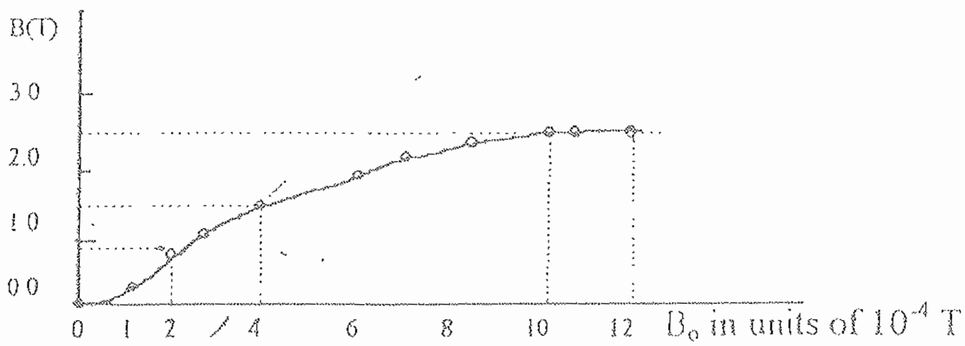
D2)- A 2500 Ω resistor is connected in series with a 100 μF capacitor, a 100 V battery and a switch. The capacitor is initially uncharged. The switch is closed at $t = 0$. At what rate is the battery delivering energy to the circuit at $t = 0.5$ sec ?

16 marks

D3)- A solenoid with 1000 turns per meter has an iron core with the magnetization curve shown in the figure below. For a B_o field of 1.2×10^{-3} T ($\mu_o = 4\pi\times10^{-7}$ N/A^2):

a)- Estimate the permeability constant κ_m ?

b)-Determine the current through the windings of the solenoid.



CHARGE AND MATTER

Problem 1:

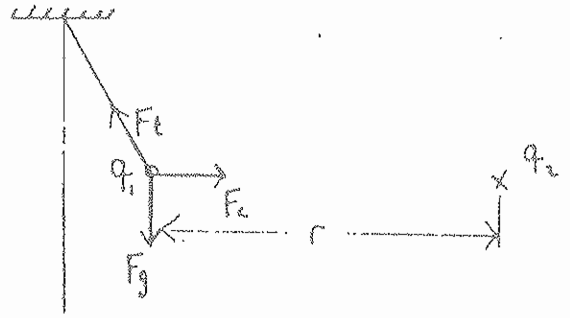
A small ball having a positive charge q_1 hangs by an insulating thread. A second ball with a negative charge $q_2 = -q_1$ is held fixed at a horizontal distance to the right of q_1.

(a) Show on a diagram the different forces acting on q_1 in its final equilibrium position. What is the magnitude of the coulomb force at this position?

(b) You are given a third ball having a charge $q_3 = 2\,q_1$. Find a position at which q_3 could be held fixed such that the first ball will hang vertically.

Solution:

(a) The diagram of different forces acting on q_1 in its final equilibrium position is shown.



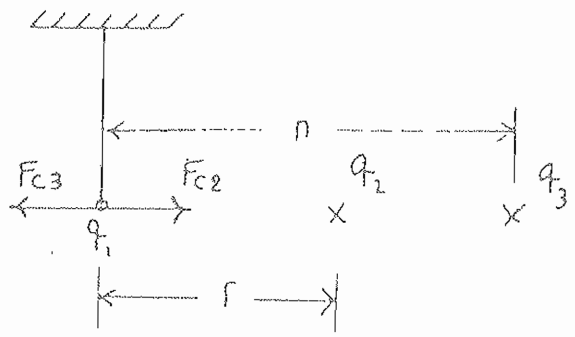
The different forces acting on q_1 are: The gravitational force (Fg), the tension force (Ft), and the coulomb force (Fc).

The coulombic force is attractional. Its magnitude is:

$$F = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1^2}{r^2}$$

(b) If q_3 is situated to the left of q_1 the thread will deflect more due to repulsion, so we situate q_3 to the right of q_1 at a distance n from it.

Besides for ease, let q_3 be on the same horizontal line as q_2 and q_1.



q_1 is in equilibrium

$\therefore\ \sum\vec{F} = 0$, therefore $\sum\vec{Fy} = o$ and $\sum\vec{Fx} = 0$

We are not interested in $\sum\vec{Fy} = o$.

$\sum\vec{Fx} = o \Rightarrow Fc_2 = Fc_3$ (look to figure)

$$\therefore\ \frac{1}{4\pi\varepsilon_0}\cdot\frac{(q_1)(q_2)}{r^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1(2q_1)}{n^2} \Rightarrow \frac{1}{r^2} = \frac{2}{n^2}.$$

$$\Rightarrow\ n = r\sqrt{2}$$

Problem 2:

Consider a charge +Q placed at the origin and two charges +q and -q placed at the points (x,y,z) = (x,a,0) and (x,-a,0) respectively. The charges q and -q are connected by a rigid rod and allowed to move in the xy plane.

Answer with justification the following two questions.

(a) Will the rod rotate? If so, how and why?

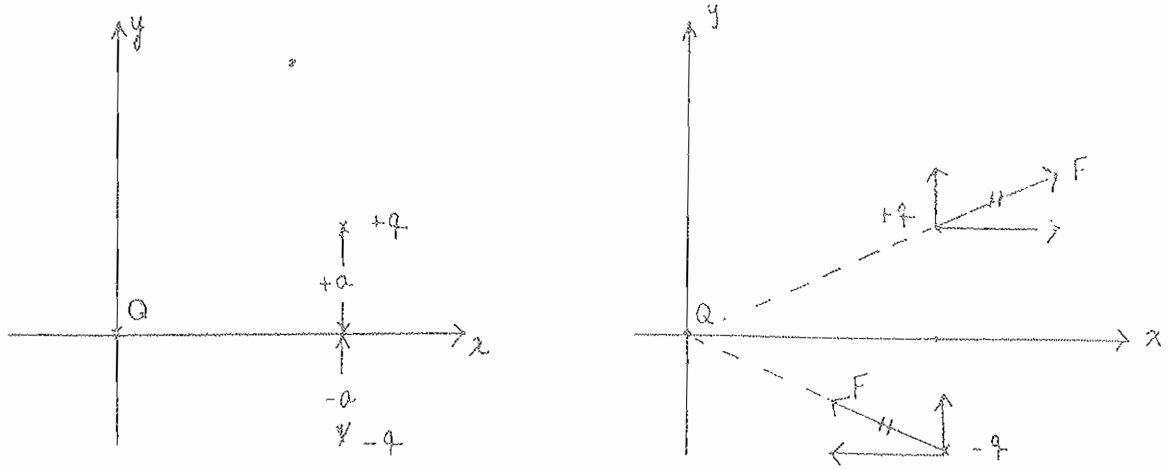
(b) Will the rod move away from or closer to Q? In either case explain why.

Solution:

(a) The two external forces are shown in the figure. If we resolve the forces to components perpendicular and parallel to rod, we observe that there is a net torque.

∴ There is rotation.

Notice that when the rod first rotates, the forces become unequal (r changes) and the motion becomes complex. As shown, the rotation is first clockwise.



(b) Again, resolving the forces we notice a net 'upward' force both in +q and -q, so we conclude that there is a net translational force at the beginning directed in the positive y direction. The rod is moving away from the origin.

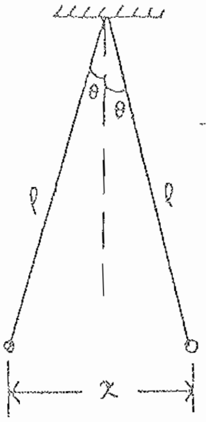
Problem 3:

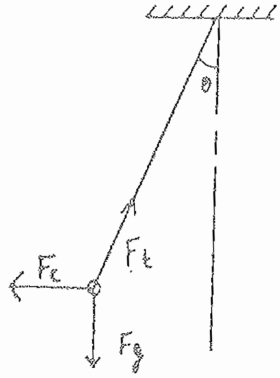
Two similar conducting balls of mass m are hung from silk threads of length l and carry similar charges q (see figure). Assume that θ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. To this approximation

(a) show that $x = \left(\dfrac{q^2 l}{2\pi \varepsilon_o mg} \right)^{1/3}$

where x is the separation between the balls.

(b) If l = 120 cm, m = 10g, and x = 5.0 cm, what is q?





Solution:

(a) The three forces acting on one of the balls are shown on the figure. The ball is in equilibrium.

$\therefore \; \sum \vec{F} = o \Rightarrow \left| \begin{array}{l} \sum \vec{F_x} = o \\ \sum \vec{F_y} = o \end{array} \right.$

$\sum \vec{F_y} = o \Rightarrow Fg = Ft \cos\theta$

$\sum \vec{F_x} = o \Rightarrow Fc = Ft \sin\theta$

Dividing the second by the first we get:

$$\frac{Fc}{Fg} = \frac{Ft \sin\theta}{Ft \cos\theta} = \tan\theta \approx \sin\theta = \frac{x}{2l} \left(\frac{opp.}{hyp.} \right)$$

$Fc = \dfrac{1}{4\pi \varepsilon_o} \cdot \dfrac{q^2}{x^2}$ and $Fg = mg$ substituting

$$\frac{q^2}{4\pi \varepsilon_o x^2 mg} = \frac{x}{2l} \Rightarrow x^3 = \frac{lq^2}{2\pi \varepsilon_o mg} \Rightarrow$$

$$x = \left(\frac{lq^2}{2\pi \varepsilon_o mg} \right)^{1/3}$$

(b) $x^3 = \frac{21q^2}{4\pi\varepsilon_0 mg} \Rightarrow q^2 = \frac{x^3 4\pi\varepsilon_0 mg}{21}$

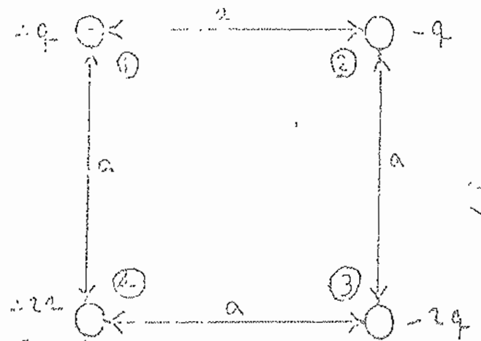
$= \frac{(0.05)^3 \times (0.0981)}{(9\times10^9) \times 2 \times 1.2} = 5.67 \times 10^{-16} \Rightarrow |q| = 2.38 \times 10^{-8}\,C$

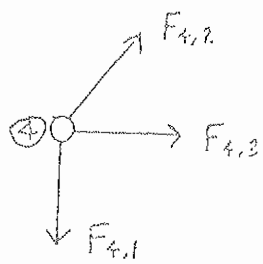
But repulsion can result either from positive or negative charges.

$\therefore \quad q = \pm 2.38 \times 10^{-8}\,C$

Problem 4:

In the figure what is the resultant force on the charge in the lower left corner of the square? Assume that $q = 1.0 \times 10^{-7}\,C$ and $a = 5.0$ cm. The charges are fixed in position.





Solution:

There are three coulombic forces acting on this corner.

$|\overrightarrow{F_{4,1}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q)(2q)}{a^2}$

$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q^2}{a^2}$ (downwards)

$|\overrightarrow{F_{4,2}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q)(2q)}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2}$

$|\overrightarrow{F_{4,2}}|_x' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2} \cdot \frac{\sqrt{2}}{2}$ (to the right)

$$|\overrightarrow{F_{4,2}}|_y = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\cdot\frac{\sqrt{2}}{2} \qquad \text{(upwards)}$$

$$|\overrightarrow{F_{4,3}}| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2q)(2q)}{a^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{4q^2}{a^2} \qquad \text{(to the right)}$$

Let us add the forces:

$$\Sigma F_x = F_{4,2x} + F_{4,3} = \frac{q^2}{4\pi\varepsilon_0 a^2}\left(\frac{\sqrt{2}}{2} + \frac{4}{1}\right) = \frac{q^2}{4\pi\varepsilon_0 a^2}\,(4.7)$$

$$\Sigma F_y = F_{4,2y} - F_{4,1} = \frac{q^2}{4\pi\varepsilon_0 a^2}\left(\frac{\sqrt{2}}{2} - 2\right) = -\frac{q^2}{4\pi\varepsilon_0 a^2}\,(1.3)$$

$$\therefore\ \vec{F} = \frac{q^2}{4\pi\varepsilon_0 a^2} \times 4.87\ \underline{|-15.4} = 0.175\ \underline{|-15.4}$$

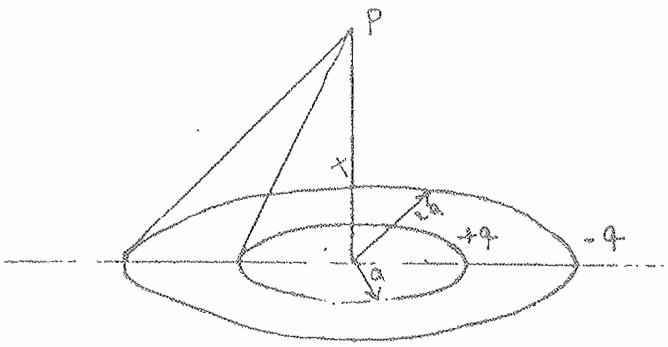
$$|\vec{F}| = 0.175\text{N}$$


$\theta = -15.4^{0}$

THE ELECTRIC FIELD

Problem 1:

Two concentric rings of radii a and 2a lying in the same plane carry equal and opposite charges q and -q respectively.



(a) Derive an expression for $\vec{E}$ at point P a distance X along the axis of the rings.

(b) What will $\vec{E}$ reduce to at large distances (X ≫ a)? What multipole does it represent?

(c) Find the potential V at P. Deduce $\vec{E}$ from it and check whether this agrees with the value obtained in (a).

Solution:

(a) $\vec{E} = \vec{E_a} + \vec{E}_{2a}$

$\vec{E_a}$: Due to symmetry, the horizontal components cancel each other.

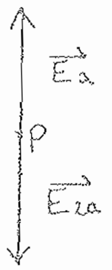
$\therefore \quad \vec{E_a} = \vec{E_x}$

$|\vec{E_{a}x}| = \int dE \cos\theta$

Where $dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2}$, $r^2 = a^2 + x^2$, and

$\cos\theta = \frac{x}{(x^2 + a^2)^{\frac{1}{2}}}$

$$\therefore \quad |\vec{E}_a| = \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq\, x}{(a^2 + x^2)^{3/2}} = \frac{x}{(a^2 + x^2)^{3/2}} \cdot \frac{1}{4\pi\varepsilon_0} \cdot q \text{ (upwards)}$$



E2a: The charges are the same but opposite by sign, therefore to find its magnitude we replace a by 2a.

$$|\vec{E_{2a}}| = \frac{x}{(4a^2 + x^2)^{3/2}} \cdot \frac{1}{4\pi\varepsilon_0} \cdot q$$

$$|\vec{E}| = |\vec{Ea}| - |\vec{E_{2a}}| = \frac{xq}{4\pi\varepsilon_0}\left(\frac{1}{(a^2 + x^2)^{3/2}} - \frac{1}{(4a^2+x^2)^{3/2}}\right)$$

(b)

$$E = \frac{xq}{4\pi\varepsilon_0} \cdot \left(\frac{1}{(a^2 + x^2)^{3/2}} - \frac{1}{(4a^2 + x^2)^{3/2}}\right)$$

$$\frac{1}{(a^2 + x^2)^{3/2}} - \frac{1}{(4a^2 + x^2)^{3/2}} = \frac{1}{x^3(1 + \frac{a^2}{x^2})^{3/2}} - \frac{1}{x^3(1 + \frac{4a^2}{x^2})^{3/2}}$$

$$= \frac{1 - 3/2\,\frac{a^2}{x^2}}{x^3} - \frac{1 - \frac{12}{2}\,\frac{a^2}{x^2}}{x^3} = \frac{9/2\,\frac{a^2}{x^2}}{x^3}$$

$$\therefore \quad E = \frac{xq}{4\pi\varepsilon_0} \cdot \frac{9}{2}\,\frac{a^2}{x^5}$$

Let $Q = 2qa^2$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{9}{4} \cdot \frac{Q}{x^4}$$

E decays proportional to $\frac{1}{x^4}$

$\therefore$ This represents a quadrupole.

(c) (To be solved after studying potentials).

Let V_1 be the electric potential due to ring of radius a at P, and let V_2 be the electric potential due to ring of radius 2a at P.

Potential at P is $V_1 + V_2$.

$$V_1 = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r_1} = \frac{q}{4\pi\epsilon_0 (a^2 + y^2)^{\frac{1}{2}}}$$

$$V_2 = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r_2} = \frac{-q}{4\pi\epsilon_0 (4a^2 + y^2)^{\frac{1}{2}}}$$

$$V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{(a^2 + y^2)^{\frac{1}{2}}} - \frac{1}{(4a^2 + y^2)^{\frac{1}{2}}} \right)$$

$$|\vec{E}| = \frac{-\partial V}{\partial y} = \frac{-q}{4\pi\epsilon_0} \left(\frac{-y}{(a^2 + y^2)^{3/2}} + \frac{y}{(4a^2 + y^2)^{3/2}} \right)$$

Problem 2 ;

(a) A uniformly charged ring of radius a and total charge (- q) lies in the X-Z plane with its center at the origin. A positive charge + 2q is placed at the center of the ring.

Find the electric field at a point P a distance y along the Y - axis.

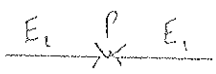
(b) What does $\vec{E}$ reduce to for large distances from the origin.

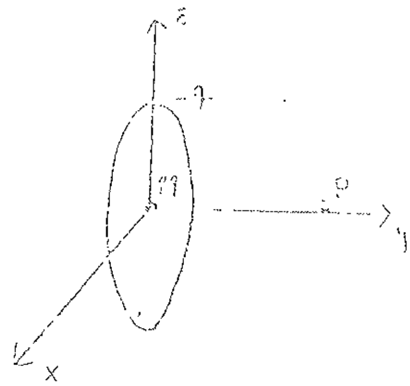
(c) Calculate the electric flux through a sphere centered at O and of radius 3 a.

Solution:

(a) $\vec{E} = \vec{E_1} + \vec{E_2}$ where $\vec{E_1}$ is the electric field created at P due to the ring and $\vec{E_2}$ is the electric field created at P due to the charge (+2q) at the origin.

Because of symmetry $\vec{E_1}$ has no component on the x or z axes. Besides, $\vec{E_1}$ points to the negative y direction because the charge on the ring is negative.





$$\vec{E_1} = \vec{E_{1y}}$$

$$E_1 = \int dE_1 \cos\theta$$

$$dE_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r^2} \quad \text{and} \quad \cos\theta = \frac{y}{(y^2 + a^2)^{\frac{1}{2}}}$$

$$\therefore E_1 = \int \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r^2}\cdot\frac{y}{(y^2 + a^2)^{\frac{1}{2}}} = \frac{yq}{4\pi\varepsilon_0(y^2 + a^2)^{3/2}}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2q)}{y^2} \quad \text{(in the positive y direction)}$$

$$\vec{E} = -\vec{E_1} + \vec{E_2}$$

$$\therefore E = \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{y^2} - \frac{y}{(y^2 + a^2)^{3/2}}\right)$$

(b) $y \gg a \Rightarrow (y^2 + a^2)^{3/2} \simeq y^3$

$$\therefore \quad = \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{y^2} - \frac{1}{y^2}\right) = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{y^2}$$

(c) $\phi = \oint \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_0} = \frac{2q-q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

(This part to be solved after studying Gauss's Law).

Problem 3:

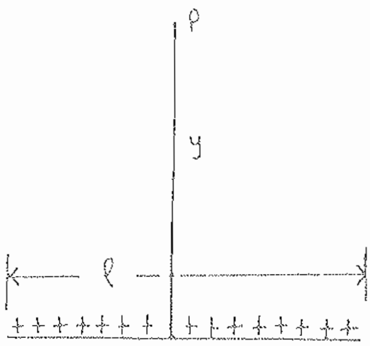
(a) A thin nonconducting rod of finite length ℓ carries a total charge q, spread uniformly along it. Show that $\vec{E}$ at a point P on the perpendicular bisector is given by

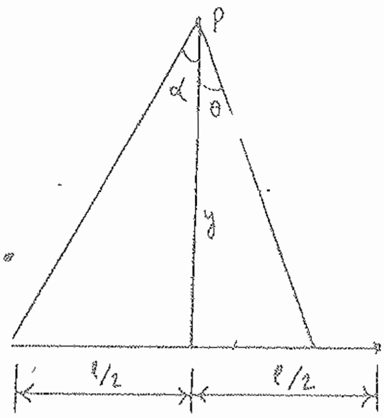
$$E = \frac{q}{2\pi\varepsilon_0 Y} \cdot \frac{1}{\sqrt{\ell^2 + 4y^2}}$$

(b) Show that as $\ell \to \infty$ this result approaches to

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}$$

(λ is the charge per unit length $= \frac{q}{\ell}$)





Solution:

(a) By symmetry $\vec{E}$ has no component on the horizontal direction.

$\vec{E} = \vec{E}_y$

$|\vec{E}| = \int dE\cos\theta$

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} \;,\quad \frac{dq}{q} = \frac{dx}{\ell} \Rightarrow \quad dq = \frac{q}{\ell}\,dx$$

$$\frac{1}{r^2} = \frac{\cos\theta^2}{y^2} \quad,\quad x = y\tan\theta \Rightarrow dx = \frac{y\,d\theta}{\cos^2\theta}$$

$$\therefore\ E = \int_{-\infty}^{+\infty} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\ell} \cdot \frac{y\,d\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{y^2} \cdot \cos\theta$$

$$= \frac{(2)(q)}{(4\pi\varepsilon_0)(\ell)(y)} \int_0^{+\infty} \cos\theta\,d\theta$$

$$\sin\alpha = \frac{\ell}{2(y^2 + \ell^2/4)^{\frac{1}{2}}} = \frac{\ell}{(4y^2 + \ell^2)^{\frac{1}{2}}} \Rightarrow \alpha = \sin^{-1}\frac{\ell}{(4y^2 + \ell^2)^{\frac{1}{2}}}$$

$$\therefore\ E = \frac{q}{(2\pi\varepsilon_0)(\ell)(y)} \cdot \sin\theta\Big|_0^{\alpha} = \frac{q}{(2\pi\varepsilon_0 y)(\ell)} \cdot \frac{\ell}{(4y^2 + \ell^2)^{\frac{1}{2}}}$$

$$= \frac{q}{2\pi\varepsilon_0 y} \cdot \frac{1}{(4y^2 + \ell^2)^{\frac{1}{2}}}$$

(b) $$(\ell^2 + 4y^2)^{\frac{1}{2}} = \ell\left(\frac{4y^2}{\ell^2} + 1\right)^{\frac{1}{2}} = \ell\left(1 + \frac{1}{2} \cdot \frac{4y^2}{\ell^2}\right) =$$

$$\ell\left(1 + \frac{2y^2}{\ell^2}\right) = \ell + \frac{2y^2}{\ell} \quad,\quad \ell \to \infty$$

$$\Rightarrow \ell + \frac{2y^2}{\ell} \longrightarrow \ell$$

$$\therefore\ E = \frac{1}{2\pi\varepsilon_0 y} \cdot \frac{q}{\ell} = \frac{\lambda}{2\pi\varepsilon_0 y}$$

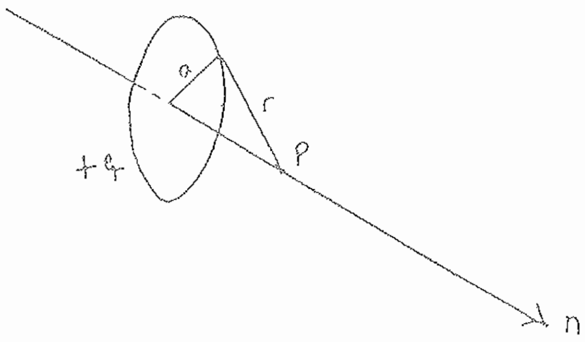
Problem 4:

An electron is constrained to move along the axis of the ring of charge (as shown on the figure). Show that the electron can perform oscillations whose frequency is given by $w = \sqrt{\dfrac{eq}{4\pi\varepsilon_0 ma^3}}$

This formula holds only for small oscillations, that is for $n \ll a$.

E at any point on the axis is given as:

$$E = \frac{qn}{4\pi\varepsilon_0 (a^2 + n^2)^{3/2}}$$



Solution:

To prove that the motion is simple harmonic it is enough to prove that F is proportional to n and opposite sign.

The second is easy and could be done qualitatively.

If n is positive (on the right side) the force is negative (pointing to the left) due to attraction of positive charges - on the ring - and the negative charge - due to the electron).

$$F = E \cdot e = \frac{qe}{4\pi\varepsilon_0} \cdot \frac{n}{(a^2 + n^2)^{3/2}}$$

If $a \gg n$ then $(a^2 + n^2)^{3/2} \longrightarrow a^3$

$\therefore \quad F = \frac{qe}{4\pi\varepsilon_0} \cdot \frac{n}{a^3}$ which is proportional to n.

$$\frac{F}{n} = \frac{qe}{4\pi\varepsilon_0 a^3} = K$$

$$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{qe}{4\pi\varepsilon_0 a^3 m}}$$

GAUSS'S LAW

Problem 1:

Electric charge Q is uniformly distributed throughout the volume of a nonconducting shell of inner radius R_1 and outer radius R_2.

(a) What is the density of charge within the shell?

(b) Use Gauss's Law, or any other method, to find the electric field at a point P a distance r from the center when:

(i) $r < R_1$

(ii) $R_1 < r < R_2$

(iii) $r > R_2$

(c) Suppose the charge Q is not uniformly distributed throughout the shell, and that the charge density is not radially ymmetric. Would Gauss's Law still hold?

Could you still use it to calculate the electric field?

Solution:

(a) Density of charge $= \sigma = \dfrac{\text{charge}}{\text{volume of shell}}$

Volume of shell $= \frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3 = \frac{4}{3}\pi (R_2^3 - R_1^3)$

$$\therefore \quad = \frac{Q}{\frac{4}{3}\pi (R_2^3 - R_1^3)} = \frac{3Q}{4\pi (R_2^3 - R_1^3)}$$

(b) (i) $r < R_1$

Using Gauss's Law

$$\oint \vec{E}.\vec{ds} = \frac{q_{net} \text{ (inside the surface)}}{\varepsilon_0}$$

But $q_{net} = 0$ for $r < R_1 \Rightarrow \oint \vec{E}\vec{ds} = 0$

$\Rightarrow \oint E.ds = 0 \Rightarrow E = 0$

(ii) $R_1 < r < R_2$

Taking a concentric sphere of radius r we get:

$$\oint \vec{E}.\vec{ds} = \frac{q\ (net)}{\varepsilon_o}$$

$\vec{E}$ and $\vec{ds}$ point to the same direction (passing through the center) $\Rightarrow \vec{E}.\vec{ds} = E.ds$

$$q_{net} = \sigma \frac{4}{3}\pi(r^3 - R_1^3) = \frac{Q(r^3 - R_1^3)}{R_2^3 - R_1^3}$$

$$\oint E.ds = \frac{q_{net}}{\varepsilon_o} = \frac{Q(r^3 - R_1^3)}{\varepsilon_o(R_2^3 - R_1^3)} = E\oint ds = E(4\pi r^2)$$

$$\therefore \quad E = \frac{Q(r^3 - R_1^3)}{4\pi r^2 \varepsilon_o (R_2^3 - R_1^3)}$$

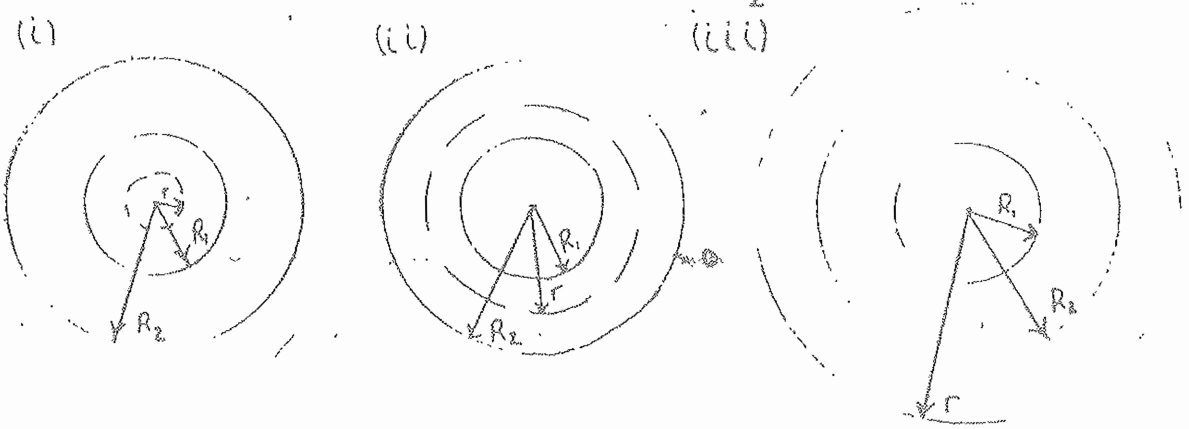
(iii) $r > R_2$

$$\oint \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_o}$$

$$\oint \vec{E}.\vec{ds} = \oint E.ds = E\oint ds = E(4\pi r^2)$$

$$\therefore \quad E(4\pi r^2) = \frac{Q}{\varepsilon_o} \Rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_o}$$

Notice that the equations at (ii) and (iii) must give the same value of E at $r = R_2$ (Boundary case).



(c) Gauss's Law still holds if the charge is not uniformly distributed, to be more specific

(i) $r < R_1$: $\vec{E} = 0$ (the same as part (b))

(ii) $R_1 < r < R_2$, we can evaluate $\vec{E}$ if we know the function of distribution ; (in this case the function is generally a function of ϕ, θ and r so it is not a function of only r) by the Gauss's Law

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}, \text{ where } q = \int \sigma \, dv.$$

(iii) $r > R_2$: $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ still holds because the net charge remains the same.

Problem 2:

Positive charge is distributed uniformly with constant density ρ throughout the volume of a sphere of radius R.

(a) Find the electric field at a point $r = \dfrac{R}{2}$.

(b) Find the electric field at a point $r = 2R$.

(c) A point charge q with mass m is released from rest at the point $r = 2R$. What is its speed v when it reaches a point $r = 3R$.

(d) What would be the total charge Q of the sphere if instead of having ρ= constant, we have $\rho = Ar$ where A = const.

Solution:

(a) Positive charge is distributed uniformly, therefore the charge is not accumulated on the surface, therefore the sphere is not made of a conducting material.

Gauss's Law: $\oint \vec{E} d\vec{s} = \dfrac{q_{net}}{\varepsilon_0}$

But both $\vec{E}$ and $\vec{ds}$ point away from the center (in the same sense because the charge is positive).

$\therefore \quad \vec{E}\vec{ds} = E.ds.\cos 0 = E.ds$

$$\oint \vec{E}.\vec{ds} = \oint E.ds = E\oint ds = E(4\pi r^2) = \frac{q_{net}}{\varepsilon_o}$$

$q_{net} = \rho(\frac{4}{3}\pi r^3) \qquad \rho =$ charge density

$$\therefore \quad E = \frac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_o(4\pi r^2)} = \frac{\rho r}{3\varepsilon_o}$$

$$\text{For } r = R/2 \Rightarrow E = \frac{\rho R}{6\varepsilon_o}$$

(b) The above equation no more applies because it is for values $o \leqslant r \leqslant R$ (inside).

$q_{net} = \rho(\frac{4}{3}\pi R^3) \qquad (\text{ if } r \geqslant R)$

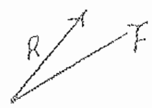
From Gauss's Law:

$$E(4\pi r^2) = \frac{q_{net}}{\varepsilon_o} = \frac{4\pi R^3}{3\varepsilon_o}$$

$$\therefore \quad E = \left.\frac{\rho R^3}{3\varepsilon_o r^2}\right|_{r = 2R} \Rightarrow E = \frac{\rho R^3}{3\varepsilon_o(4R^2)} = \frac{\rho R}{12\varepsilon_o}$$

(a)

(b)



(c) (This part to be solved after studying the concepts of potential and conservation of energy).

This part will be solved in two methods.

First Method: In the initial and final positions the charge is outside the sphere so the sphere can be taken as a point charge located at the center having a charge $Q = \rho(\frac{4}{3}\pi R^3)$.

The potential (internal) energy at r = 2R is:

$$W_1 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q.Q}{(2R}$$

The potential (internal) energy at r = 3R is

$$W_2 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{qQ}{(3R)}$$

If the charge goes from r = 2R to r = 3R the change of potential (internal) energy is:

$$\Delta W = W_2 - W_1 = \frac{qQ}{4\pi\varepsilon_o}\left(\frac{1}{3R} - \frac{1}{2R}\right) = -\frac{qQ}{4\pi\varepsilon_o} \cdot \frac{1}{6R}$$

Substituting $Q = \rho(\frac{4}{3}\pi R^3)$ we get

$$\Delta W = -\frac{q\rho R^2}{18\varepsilon_o}$$

The kinetic energy is increased by the same amount of that decreased potential (internal) energy.

$\therefore \quad |\Delta W| = |\Delta K.E.|$

but the initial kinetic energy is zero.

$$\therefore \quad K.E._2 = \frac{1}{2}mV_2^2 = \frac{q\rho R^2}{18\varepsilon_o} \Rightarrow V_2^2 = \frac{q\rho R^2}{9m\varepsilon_o}$$

$$\therefore \quad V_2 = \frac{R}{3}\left(\frac{q\rho}{m\varepsilon_o}\right)^{1/2}$$

<u>Second Method:</u>

$|\Delta W|$ = change of potential (internal energy)

$$= \int \vec{F}.\vec{dx} = \int q\vec{E}\vec{dx} = +q\int \vec{E}\vec{dr} = q\int E.dr$$

$$= q\int \frac{\rho R^3}{3\varepsilon_o r^2} \cdot dr = \frac{q\rho R^3}{3\varepsilon_o} \int \frac{1}{r^2} dr = \frac{q\rho R^3}{3\varepsilon_o} \left| -\frac{1}{r} \right|_{2R}^{3R}$$

$$= \frac{q\rho R^3}{3\varepsilon_o} \left(\frac{1}{2R} - \frac{1}{3R} \right) = \frac{q\rho R^2}{18\varepsilon_o}$$

After this point the continuation is like the first method (i.e. this energy is converted to K.E. and since initial K.E. $= 0 \therefore \frac{q\rho R^2}{18\varepsilon_o} = \frac{1}{2} m V_2^2 \Rightarrow V_2 = \frac{R}{3}\left(\frac{q\rho}{m\varepsilon_o}\right)^{\frac{1}{2}}$).

(d) We have always:

$$Q = \int_0^R \rho \, dV$$

Taking spherical coordinates

$$dV = 4\pi r^2 dr$$

$$Q = \int_0^R (Ar)(4\pi r^2 dr) = 4\pi A \int_0^R r^3 dr$$

$$= 4\pi A \left(\frac{R^4}{4}\right) = \pi A R^4$$

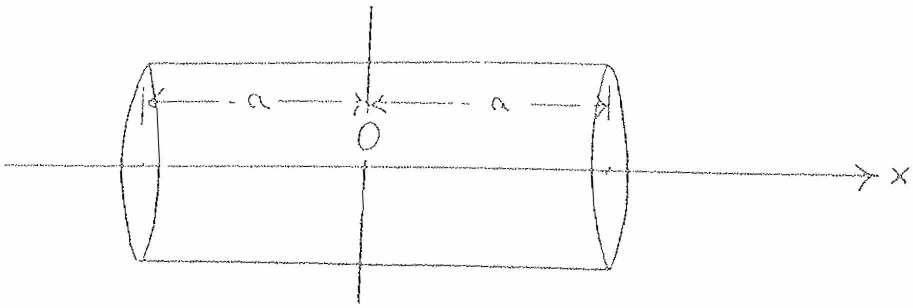
Problem 3:

Let the electric field $\vec{E} = E(x)\vec{i} = (x^3 + x^2)\vec{i}$ where x is the distance from the origin 0 and $\vec{i}$ is the unit vector along the x-axis.

Consider a cylinder of length 2a and radius r lying along the x-axis from $x = -a$ to $x = +a$.

(a) Find the flux of the elctric field 1) through the right-hand cap, 2) through the left-hand cap, and 3) through the lateral surface of the cylinder.

(b) What is the charge enclosed within the cylinder?



Solution:

(a) $\phi = \oint \vec{E}.\vec{ds} = \vec{Ex}\vec{ds}_x + \vec{Ey}\vec{dsy} + \vec{Ez}\vec{dsz} = \vec{Ex}\vec{ds}_x$

Because Ey = 0 and Ez = 0

1. Right-hand cap: (x = a)

$\vec{E} = (x^3 + x^2)\vec{i} = (a^3 + a^2)\vec{i} \Rightarrow Ex = a^3 + a^2$

$\therefore \phi_1 = \int (a^3 + a^2) ds = (a^3 + a^2) \int ds = (a^3 + a^2) 4\pi r^2$

2. Left-hand cap:

$E = (x^3 + x^2) = (-a^3 + a^2)$ (x = -a)

$\vec{E}.\vec{ds} = E.ds.\cos 180^o = -E.ds$ (ds outwards while E inwards). (In the first case both were directed outwards).

$\therefore \phi_2 = -\int (-a^3 + a^2) ds = (a^3 - a^2) \int ds = (a^3 - a^2)(4\pi r^2)$

3. Lateral surface:

$\phi = \int \vec{Ex}\vec{ds}x$ but $ds_x = 0$ (no component in the x-axis because it is on the y-z plane)

$\therefore \phi_3 = 0$

(b) $\phi = \phi_1 + \phi_2 + \phi_3 = 4\pi r^2(a^3 + a^2 + a^3 - a^2)$

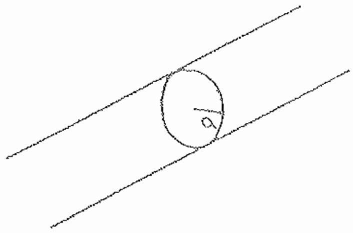
$= 4\pi r^2(2a^3) = 8\pi r^2 a^3$

$\phi = \int \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_o} \Rightarrow q_{net} = \phi\varepsilon_o$

$\therefore$ $q_{net} = (8\pi r^2 a^3)\varepsilon_o = 8\varepsilon_o\pi r^2 a^3$

Problem 4:

Consider an infintely long non-conducting cylinder of radius a and of uniform charge density per unit length . Find the electric field E a distance r from the axis of the cylinder both for r < a and for r > a.



Solution:

For r < a.

As a Gaussian surface we chose a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis.

$$\oint \vec{E}.\vec{ds} = \oint E.ds = E\oint ds = E(2\pi rh)$$

$$q = (\lambda h)(\frac{r^2}{R^2}) \quad \text{But} \oint E.ds = \frac{q_{net}}{\varepsilon_o}$$

$$\therefore \quad E(2\pi rh) = \frac{\lambda hr^2}{\varepsilon_o R^2}$$

$$\therefore \quad E = \frac{\lambda r}{2\varepsilon\pi R^2}$$

We notice that E is proportional to r.

For $r > a$.

As a Gaussian surface we chose a circular cylinder of radius r and length h, closed at each end of plane caps normal to the axis.

$$\oint \vec{E}.\vec{ds} = \oint E.ds = E\oint ds = E(2\pi rh)$$

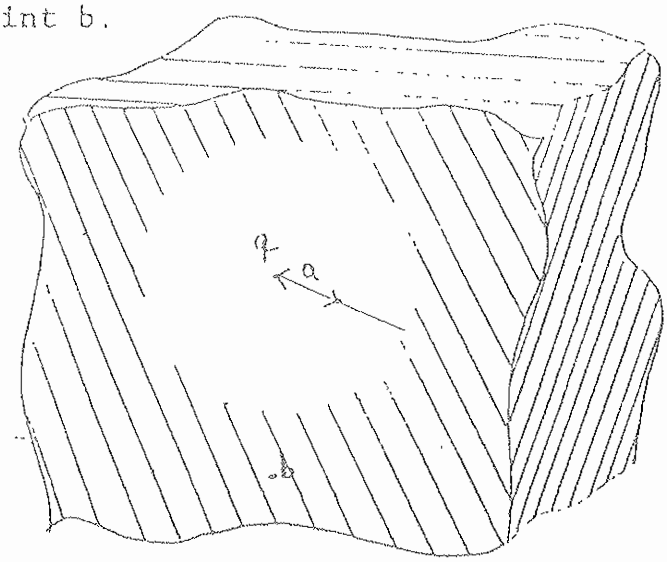
$$q = \lambda h \quad \text{But} \oint \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_0}$$

$$\therefore \quad E(2\pi rh) = \frac{\lambda h}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi r\varepsilon_0}$$

We must notice two things. First, E is inversely proportional with r. Second, the two expressions of E (for $r < a$ and $r > a$) must give the same answers at $r = R$ because of boundary conditions.

Problem 5:

The figure shows a point charge of 1.0×10^{-7}C at the center of a spherical cavity of radius 3.0 cm in a piece of metal. Use Gauss's law to find the electric field (a) at point a, halfway from the center to the surface and (b) at point b.



Solution:

(a) For the Gaussian surface take a concentric surface with $r = \frac{1}{2}a$.

$$\oint \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_o}$$

$$q_{net} = Q = +3.0 \times 10^{-6} C$$

$\vec{E}$ and $\vec{ds}$ have the same sense (both away from the center)

$$\therefore \quad \vec{E}.\vec{ds} = E.ds$$

$$\therefore \quad \oint \vec{E}.\vec{ds} = \oint E.ds = E\oint ds = E(4\pi r^2) = \frac{q_{net}}{\varepsilon_o}$$

$$\therefore \quad E = \frac{(q_{net})}{\varepsilon_o(\pi a^2)} = \frac{(1 \times 10^{-7})}{(8.85 \times 10^{-12})(\pi)(0.03)^2} = 4 \times 10^6 \text{ N/C}$$

(b) For a Gaussian surface take a concentric surface with $r > a$.

$$\oint \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_o}$$

$$q_{net} = Q - Q = 0$$

The explanation is that the system is in equilibrium (at low energy) so the positive charge attracts the same quantity of negative charge (-Q) on the surface of the sphere (r=a). So, at any distance r>a, the net charge is zero.

$$\oint \vec{E}.\vec{ds} = 0 \Rightarrow E\oint ds = 0 \Rightarrow E = 0$$

Problem 1:

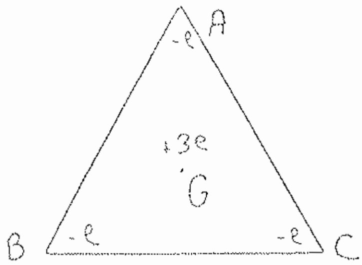
(a) Three electrons are brought from infinity to the corners of an equilateral triangle of side 10 cm. Calculate the work done.

(b) A nucleus of charge +3e is then brought from infinity to the center of this triangle. Calculate the work done.

(c) One of the electrons is released and accelerates towards the nucleus. Find the velocity of impact.

Solution:

(a) The total work done is stored in the system ABC.



$$U_{T1} = U_{AB} + U_{AC} + U_{BC}$$

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_2}$$

$$U_{AB} = \frac{(9 \times 10^9).(-1.6 \times 10^{-19}).(-1.6 \times 10^{-19})}{10 \times 10^{-2}} = 2.304 \times 10^{-27} J$$

similarly

$U_{AC} = 2.304 \times 10^{-27} J$ and

$U_{BC} = 2.304 \times 10^{-27} J$

$$\therefore U_{T1} = 3 \times (2.30 \times 10^{-27}) = 6.912 \times 10^{-27} J$$

(b) $U_{T2} = U_{AG} + U_{BG} + U_{CG}$

But $U_{AG} = U_{BG} = U_{CG} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{(3e)(-e)}{AG}$

$$= \frac{(9 \times 10^9)(3 \times 1.6 \times 10^{-19})(-1.6 \times 10^{-19})}{(0.0577)} = -1.198 \times 10^{-26} J$$

$\therefore \quad U_{T2} = 3 \times U_{AG} = -3.594 \times 10^{-26} J$

(c) Initially the energy in the system is:

$U = U_{T1} + U_{T2} = 6.912 \times 10^{-27}$

$- 3.593 \times 10^{-26} = -2.9 \times 10^{-26} J$

Finally the energy in the system is:

$U_T^1 = U_{BG}^1 + U_{CG}^1 + U_{BC}$

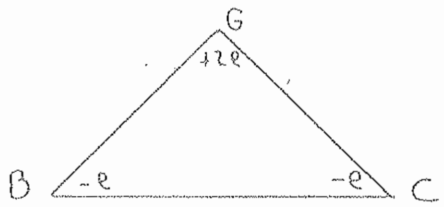
$U_{BG}^1 = U_{CG}^1 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{(+2e)(-e)}{BG}$

$= -8 \times 10^{-27} J$

$U_{BC} = 2.304 \times 10^{-27} J$

$\therefore \quad U_T^1 = -16 \times 10^{-27} + 2.3 \times 10^{-27} = -13.7 \times 10^{-27} J$

$\Delta E = U_T^1 - U_T = -13.7 \times 10^{-27} + 2.9 \times 10^{-26} = 1.53 \times 10^{-26} J$



But because of conservation of total energy this change in potential energy must have been converted to K.E.

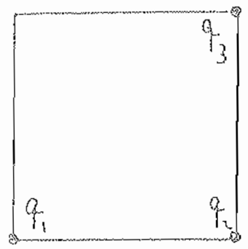
$\therefore \quad K.E. = \frac{1}{2}mV^2 = 1.53 \times 10^{-26}$

$\therefore \quad V^2 = \frac{1.53 \times 10^{-26} \times 2}{9.11 \times 10^{-31}} = 33590 m^2/s^2$

$\therefore \quad V = 183 \; m/s$

Problem 2:

Three point charges $q_1 = 5C$, $q_2 = -10C$ and $q_3 = 2C$ are placed at the corners of a square of side a = 10 cm as shown.



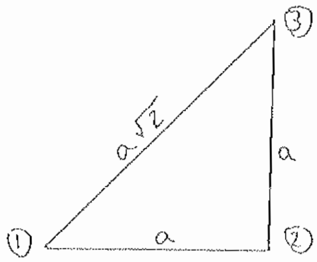
(a) The energy U stored in the system is ...?

(b) If the charge q_3 is now moved to the other empty vertex of the square, how much work is done on (or by) the charge q_3?

Solution:

(a) Energy stored in the system:

$$U = U_{12} + U_{23} + U_{13}$$



$$U_{12} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{a} = (9 \times 10^9) \times \frac{(5)(-10)}{(0.1)} = -4.5 \times 10^{12} J$$

$$U_{23} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_2 q_3}{a} = \frac{(9 \times 10^9)(-10)(2)}{(0.1)} = -1.8 \times 10^{+12} J$$

$$U_{13} = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_3}{(a\sqrt{2})} = \frac{(9 \times 10^9).(5)(2)}{\sqrt{2} \times 0.1} = 6.36 \times 10^{11} J$$

$$\therefore \quad U = -4.5 \times 10^{12} - 1.8 \times 10^{12} + 6.36 \times 10^{11}$$

$$= -5.64 \times 10^{12} J$$

(b) Let U = Total energy stored in the new system.

$$U^1 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{1}{a} \left(q_3 q_1 + q_1 q_2 + \frac{q_3 q_2}{\sqrt{2}}\right)$$

$$= (9 \times 10^9)(10) \cdot \left(5 \times 2 - 10 \times 5 \cdot \frac{2 \times 10}{\sqrt{2}}\right)$$

$$= -4.879 \times 10^{12} J$$

$$\Delta U = W = U^1 - U = 7.9 \times 10^{11} J$$

Problem 3:

The potential at a distance d from a point charge is 600 V and the magnitude of the electric field is 200 N/C.

(a) What is the distance to the point charge?

(b) How much work is done on or by a charge of -3 coulombs to be moved from a distance of 5d to the point d above? Explain.

(c) Four positive charges of 2C each are placed at the corners of a rectangle of 3 x 4 cm. What is the energy stored in the system?

Solution:

(a) For a point charge:

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \quad \text{and} \quad V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$$

$$\therefore \quad \frac{V}{r} = E \rightarrow r = \frac{V}{E} = \frac{600}{200} = 3m$$

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r} \Rightarrow q = rV(4\pi\varepsilon_o) = \frac{(3)(600)}{(9 \times 10^9)}$$

$$= 2 \times 10^{-7} C$$

(b) The quantity of work done = U′ - U where U′ and U are the potential energies at the final and the initial positions respectively.

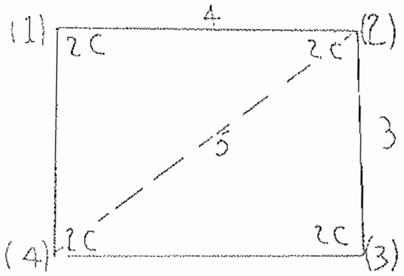
$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{5d'} = \frac{(9 \times 10^9)(2 \times 10^{-7})(-3)}{5 \times 3} = 360J$$

$$U' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{d} = \frac{(9 \times 10^9)(2 \times 10^{-4})(-3)}{3} = -1800J$$

$W = U^1 - U = -1800 + 360 = -1440J$

The work done is negative indicating that work is done by the charge so that if we let the charge free it will do work (positive) due to conservation of energy.

(c) Let U be the total energy stored in the system.



$$U = U_{12} + U_{34} + U_{23} + U_{14} + U_{13} + U_{24}$$

$$= \frac{q^2}{4\pi\varepsilon_0 10^{-2}}\cdot\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3} + \frac{1}{3} + \frac{1}{5} + \frac{1}{5}\right)$$

$$= (2)^2(9 \times 10^{11})(1.566) = 5.64 \times 10^{12}J$$

<u>Problem 4 :</u>

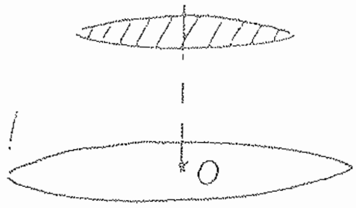
(1) Find the potential at the center of a hemispherical shell of radius R = 10 cm and charge Q = 3 c (the shell is charged uniformly and is a <u>non</u> conductor). Do this from first principles .

(2) What would the potential be at the center of a spherical shell one half of which has uniform charge + Q and the other half a uniform charge - 2Q. Explain .

(3) How much work is done on or by a charge q = 2 C to bring it from infinity to the center of the sphere of part 2 .

Solution:

(1) The shell is a non-conductor, therefore the charge is not distributed only on the surface.



$$\rho = \text{charge density} = \frac{\text{charge}}{\text{volume}} = \frac{Q}{4/6\pi R^3}$$

$$= \frac{3 \times 6}{4\pi(0.1)^3} = 1432.4\ c/m^3$$

$$V = \int dV = \frac{1}{4\pi\varepsilon_o} \cdot \frac{dq}{r} = \int \frac{1}{4\pi\varepsilon_o} \cdot \frac{\rho dV}{r}$$

where $dV = \pi r^2 dr$ $(0 < r < R)$

$$\therefore\ V = \frac{\rho}{4\varepsilon_o}\int r dr = \frac{\rho}{4\varepsilon_o}\left|\frac{r^2}{2}\right|_0^R$$

$$= \frac{\rho}{4\varepsilon_o} \cdot \frac{R^2}{2} = \frac{R^2}{8\varepsilon_o} = 2.02 \times 10^{11} V$$

(2) $V = V_1 + V_2$ (added algebraically) where V_1 is the potential at the center due to $+Q$ and V_2 is the potential at the center due to $-2Q$.

$V_1 = 2.02 \times 10^{11} V$ (calculated in (a))

$V_2 = -2 \times V_1 = -4.04 \times 10^{11} V$ (because Q is related to ρ and ρ is related to V linearly).

$\therefore\ V = 2.02 \times 10^{11} - 4.04 \times 10^{11} = -2.02 \times 10^{11} V.$

(3) The potential at the center of the sphere is known so

$$W = q.V = (2)(-2.02 \times 10^{11}) = -4.04 \times 10^{11} J$$

Work is negative indicating that work is done by the charge.

Problem 5:

Two point charges +q and -2q are a distance d apart.

Determine the equipotential surface with V = o

Solution:

Let us find the potential created by each of the charges at any point.

$$V_1 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r_1}$$

$$V_2 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{(-2q)}{r_2}$$

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0$$

$$\therefore \quad \frac{1}{r_1} = \frac{2}{r_2} = \frac{r_2}{r_1} = 2$$

Let A be the variable point satisfying this equality.

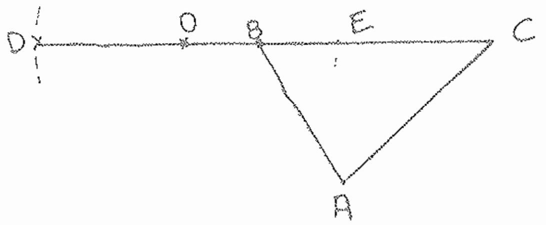
$$\therefore \quad \frac{AC}{AB} = 2$$

From A draw the internal and external bisectors of $\triangle ABC$.

The equipotential surface is a sphere whose diamter (DE) is the intersections of the bisectors with the line BC.

$DE = DB + BE = d + d/3 = 4d/3 \Rightarrow R = \frac{2d}{3}$

$\Rightarrow \quad OB = BD - OD = d - \frac{2d}{3} = \frac{d}{3}$ (center located)



Problem 6:

Three particles having charges of +Q, +Q and -2Q are located at the corners of an equilateral triangle of edge length L.

(a) How much work must be done to completely separate the three particles from one another?

(b) If the three particles are held fixed at the corners of the triangle, how much work must be done in bringing a charge -Q from infinity to the center of the triangle?

Solution:

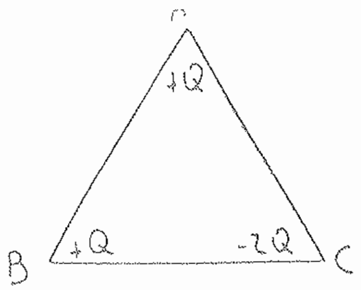
(a) The magnitude of work done to separate the 3 particles is equal to the magnitude of work done to bring these particles in this order from infinity.

The energy (potential) found in the system is:

$$U_T = U_{AB} + U_{AC} + U_{BC}$$

$$= \frac{Q^2}{4\pi\varepsilon_o L}(1-2-2)$$

$$= -\frac{3 \cdot Q^2}{4\pi\varepsilon_o L}$$



(for more detailed explanation refer to problem 1 or 2)

But the work done to separate the 3 particles is opposite in sign to the work done to bring these particles together.

$$\therefore \quad W = -U_T = + \frac{3Q^2}{4\pi\varepsilon_o L}$$

(b) (For the sake of not repeating ourselves, this part will be solved in a slightly different way than problem 1, part (b)).

$$VO = V_{OC} + V_{OB} + V_{OC}$$

$$= \frac{Q}{4\pi\varepsilon_o OA}(1 + 1 - 2) = 0$$

$$\text{where } OA = OB = OC = \frac{L}{\sqrt{3}}$$

The work done is proportional to the potential of the initial and final positions but both of them are zero, therefore the work done is zero. Mathematically:

$$W = qV = (-Q).(O) = 0$$

Note: Initially the charge -Q is assumed to be at infinity having a zero potential.

GENERAL PROBLEMS

Problem 1:

Three charges are placed as follows:

$q_1 = 10^{-6}$C. at x = 0, y = 0, z = 0

$q_2 = 3 \times 10^{-6}$C. at x = 3cm, y = 3, cm z = 0

$q_3 = -5 \times 10^{-6}$C. at x = 0, y = 0, z = 2cm

(a) Find the energy stored in the system.

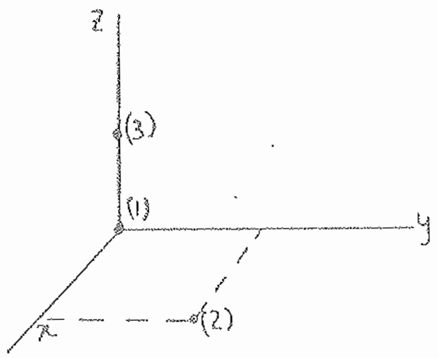
(b) Find the potential at a point (x,y,z) in space.

(c) Find the electric field at a point (x,y,z) in space.

(d) Find the flux of the electric field over a sphere centered at the origin and of radius 2.5 cm.

Solution:

(a) Let U be equal to the energy (potential) stored in the system.



$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\varepsilon_o} \cdot \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1.0^{-10}}{4\pi\varepsilon_o} \cdot \left(\frac{(1)(3)}{(4.243)} + \frac{(1)(-5)}{(2)} + \frac{(3)(-5)}{(4.69)}\right)$$

$$= (9 \times 10^9) \cdot (10^{-10}) \cdot (-5) = -4.5 \text{ J}$$

(b) We will find the potential at (x,y,z) by each of the charges and then will superimpose them.

$$V = V_1 + V_2 + V_3$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \right)$$

$$r_1 = (x^2 + y^2 + z^2)^{1/2} \qquad \text{(cm)}$$

$$r_2 = \left[(x-3)^2 + (y-3)^2 + z^2\right]^{1/2} \qquad \text{(cm)}$$

$$r_3 = \left[x^2 + y^2 + (z-2)^2\right]^{1/2} \qquad \text{(cm)}$$

$$\therefore \quad V = (9 \times 10^5) \cdot \left(\frac{1}{(x^2+y^2+z^2)^{1/2}} + \frac{3}{\left[(x-3)^2 + (y-3)^2 + z^2\right]^{1/2}} - \frac{5}{\left[(x^2 + y^2 + (z-2)^2\right]^{1/2}} \right)$$

(c) We cannot treat with $\vec{E}$ as easily as in V because $\vec{E}$ is a vector and algebraic addition is not possible. However we can escape from these complicated calculations by inducing $\vec{E}$ from V. Thus -

$$Ex = -\frac{\partial V}{\partial x}, \quad Ey = -\frac{\partial V}{\partial y} \quad \text{and} \quad Ez = -\frac{\partial V}{\partial z}$$

$$Ex = -\left[\frac{-x}{(x^2 + y^2 + z^2)^{3/2}} + \frac{-(x-3)3}{\left[(x-3)^2 + (y-3) + z^2\right]^{3/2}} + \frac{-x(-5)}{\left[x^2 + y^2 + (z-2)^2\right]^{3/2}} \right]$$

$$= \frac{x}{(x^2 + y^2 + z^2)^{3/2}} + \frac{3 \cdot (x-3)}{\left[(x-3)^2 + (y-3)^2 + z^2\right]^{3/2}} - \frac{5 \cdot x}{\left[x^2 + y^2 + (z-2)^2\right]^{3/2}}$$

$$E_y = \frac{y}{(x^2+y^2+z^2)^{3/2}} + \frac{3\cdot(y-3)}{[(x-3)^2+(y-3)^2+z^2]^{3/2}} - \frac{5\cdot y}{[x^2+y^2+(z-2)^2]^{3/2}}$$

$$E_z = \frac{z}{(x^2+y^2+z^2)^{3/2}} + \frac{3\cdot z}{[(x-3)^2+(y-3)^2+z^2]^{3/2}} - \frac{5\cdot(z-2)}{[x^2+y^2+(z-2)^2]^{3/2}}$$

$$\vec{E} = \vec{E}x + \vec{E}y + \vec{E}z$$

(d) $\phi = \int \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_o} = \frac{q_1 + q_3}{\varepsilon_o} = -\frac{4 \times 10^{-6}}{8.85 \times 10^{-12}}$

$= -4.52 \times 10^5$ Wb.

Notice that q_2 is outside the sphere.

Problem 2:

A uniform charge density of ρ C/m^3 is in the shape of a sphere of radius a. This sphere is surrounded by a thin spherical conducting shell of radius b. The shell has a total charge of Q coulomb on it.

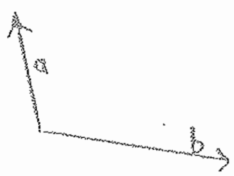
Find the electric field and potential at a radius r from the center of the sphere, where ..

a) $r < a$

b) $a < r < b$

c) $r > b$

d) $r = b$



Solution:

First we will find the electric fields and then the potentials.

Using Gauss's law and taking the sphere centered at O and radius equal to r:

$$\oint \vec{E}\vec{ds} = \frac{q_{net}}{\varepsilon o}$$

$$\oint \vec{E}\vec{ds} = \oint Eds = E\oint ds = E(4\pi r^2)$$

(a) $q_{net} = \rho(\frac{4}{3}\pi r^3)$

(b) $q_{net} = \rho(\frac{4}{3}\pi R^3)$

(c) $q_{net} = \rho(\frac{4}{3}\pi R^3) + Q$

(d) $q_{net} = \rho(\frac{4}{3}\pi R^3)$

(a) $r < a$:

$$E = \left(\frac{4\rho\pi r^3}{3\varepsilon o}\right)\cdot\left(\frac{1}{4\pi r^2}\right) = \frac{\rho r}{3\varepsilon o}$$

(b) $r < a < b$:

$$E = \left(\frac{4\rho\pi R^3}{3\varepsilon o}\right)\cdot\left(\frac{1}{4\pi r^2}\right) = \frac{\rho R^3}{3\varepsilon o r^2}$$

(c) $r > b$:

$$\therefore \quad E = \left(\frac{4\rho\pi R^3}{3\varepsilon o} + \frac{Q}{\varepsilon o}\right)\cdot\left(\frac{1}{4\pi r^2}\right)$$

$$= \frac{(4\rho\pi R^3 + 3Q)}{12\varepsilon o\pi r^2}$$

(d) $r = b$:

Notice that Q is not included because both on the sphere and on the shell the charges are positive and try to repel each other. Besides, the shell is conducting so the charges in it are distributed on the outer surface of the shell.

$$\therefore \quad E = \frac{4\rho\pi R^3}{\varepsilon_o 3(4\pi r^2)} = \frac{\rho R^3}{3\varepsilon_o r^2} \qquad \text{(like (b))}$$

Let us find first V for r>b.

$$V = -\int_{\infty}^{r} \vec{E}\cdot\vec{dl} = +\int E dl = -\int E dr$$

$$= -\int\left(\frac{4\rho\pi R^3 + 3Q}{12\varepsilon_o\pi r^2}\right)dr = -\left(\frac{4\rho\pi R^3 + 3Q}{12\varepsilon_o\pi}\right)\cdot\left|-\frac{1}{r}\right|_{\infty}^{r}$$

$$= +\frac{4\rho\pi R^3 + 3Q}{12\varepsilon_o\pi}\cdot\frac{1}{r}$$

V at r = b.

$$V = -\int\vec{E}\cdot\vec{dl} = -\int_{\infty}^{b} E\,dr = -\int_{\infty}^{b+} E dr - \int_{b+}^{b} E dr \to 0$$

$$= \frac{4\rho\pi R^3 + 3Q}{12\varepsilon_o\pi}\cdot\frac{1}{b}$$

V at a<r<b.

$$V = -\int\vec{E}\vec{dl} = -\int E dr = -\int_{\infty}^{b^+} E dr - \int_{b^+}^{r} E dr$$

$$= \frac{4\rho\pi R^3 + 3Q}{12\varepsilon_o\pi b} - \int_{b^+}^{r}\left(\frac{\rho R^3}{3\varepsilon_o r^2}\right)\cdot dr$$

$$= \frac{4\rho\pi R^3 + 3Q}{12\varepsilon_o\pi b} + \frac{\rho R^3}{3\varepsilon_o}\cdot\left(+\frac{1}{r}\right)\Bigg|_{b}^{r}$$

$$= \frac{4\rho\pi R + 3Q}{12\varepsilon_o\pi b} - \frac{\rho R^3}{3\varepsilon_o b} + \frac{\rho R^3}{3\varepsilon_o r}$$

$$= \frac{\rho R^3}{3\varepsilon_o r} + \frac{Q}{4\varepsilon_o\pi b}$$

V at a $= \dfrac{\rho R^3}{3\varepsilon_o a} + \dfrac{Q}{4\varepsilon_o \pi b}$

V at r<a.

$$V = -\int \vec{E}\,d\vec{l} = -\int_{\infty}^{r} E\,dr = -\int_{\infty}^{a} E\,dr - \int_{a}^{r} E\,dr$$

$$= \left(\frac{\rho R^3}{3\varepsilon_o a} \quad \frac{Q}{4\varepsilon_o \pi b}\right) - \int_{a}^{r} \left(\frac{\rho r}{3\varepsilon_o}\right) \cdot dr$$

$$= \quad \ldots \quad - \frac{\rho}{3\varepsilon_o}\left(\frac{r^2}{2}\right)_a^r$$

$$= \quad \ldots \quad - \frac{\rho}{3\varepsilon_o} \cdot \frac{r^2}{2} + \frac{\rho}{3\varepsilon_o} \cdot \frac{a^2}{2}$$

$$\therefore \quad V = \left(\frac{\rho R^3}{3\varepsilon_o a} + \frac{Q}{4\varepsilon_o \pi b} + \frac{a^2}{6\varepsilon_o}\right) - \frac{\rho r^2}{6\varepsilon_o}$$

Notice that usually we cannot integrate V directly because E has different functions in different regions.

Problem 3:

The volume density of charge within a long circular cylinder of radius R is given by: $\rho = cr$ $(r < R)$ where c is a constant and r the distance from the axis of the cylinder.

(a) What is the total charge in a length L of the cylinder?

(b) What is the electric field at a point within the cylinder a distance r $(r < R)$ from the axis?

(c) The potential within the cylinder is given by:
$V = -\dfrac{cr^3}{6\varepsilon_o} + k$ where k is a constant. Show how you can

obtain the value of the electric field from the expression for the potential.

(a) $Q = \int_0^L \rho\, dV = \int (cr)(\;R^2 dr)$

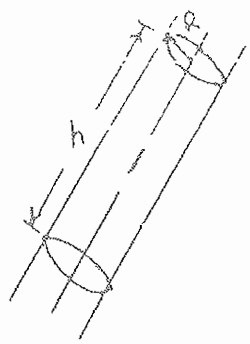
$$= c\pi R^2 \left(\frac{r^2}{2}\right)_0^L = \frac{c\pi R^2 L^2}{2}$$

(b) For the Gaussian surface we chose a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis.

$$\oint \vec{E}\,\vec{ds} = \oint E\, ds = E \oint ds = E(2\pi r h)$$

$$q_{net} = \rho\,.\,\text{volume}$$

$$= (cr)\,.\,(\pi r^2 h) = c\pi h r^3$$



but $\oint \vec{E}\,\vec{ds} = \dfrac{q_{net}}{\varepsilon_0}$

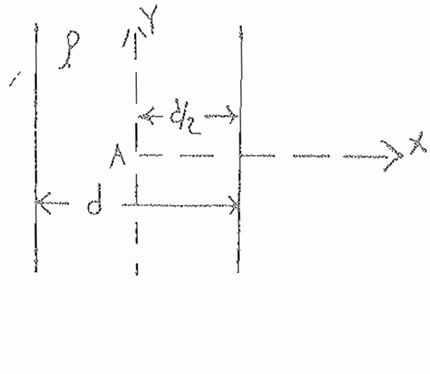
$$\therefore \quad E(2\pi r h) = \frac{(c\pi h r^3)}{\varepsilon_0}$$

$$\therefore \quad E = \frac{cr^2}{2\varepsilon_0}$$

(c) $E = \dfrac{-dV}{dr} = -V'_r = -\left(\dfrac{-cr^3}{6\varepsilon_0} + k\right)' = \dfrac{cr^2}{2\varepsilon_0} = E$

Problem 4:

A large nonconducting plane slab of thickness d has a uniform volume charge density ρ.



(a) Find the magnitude of the electric field at a distance x from the plane bisecting the slab. For $x < \frac{d}{2}$ and $x > \frac{d}{2}$.

(b) Calculate the potential difference between the point A at $x = 0$, $y = 0$ and the point B at $x = \frac{d}{2}$, $y = d$.

Solution:

(a) Let us take the Gaussian surface the closed cylinder of cross-sectional area A and height 2x, the cross-sectional area being parallel to the plane slab. From symmetry $\vec{E}$ points at right angles to the end caps and away from the plane. $\vec{E}$ does not pierce the cylindrical surface, therefore there is no contribution to the flux from this source. Applying Gauss's Law:

$$\frac{q_{net}}{\varepsilon_0} = \oint \vec{E}\vec{ds} = \oint E.ds = E \oint ds$$

$$= E(A + A) = 2EA.$$

$x < \frac{d}{2}$: $q = \rho(A.2x)$

$x > \frac{d}{2}$: $q = \rho(A.d)$

$\therefore$ $x > \frac{d}{2}$: $E = (\frac{\rho A.2x}{\varepsilon_0}) . (\frac{1}{2A}) = \frac{\rho x}{\varepsilon_0}$

$\therefore$ $x > \frac{d}{2}$: $E = (\frac{\rho A d}{\varepsilon_0}) \cdot (\frac{1}{2A}) = \frac{\rho d}{2\varepsilon_0}$

(b) Because of symmetry we can move freely in the y-direction without changing the electric field in that direction.

$\therefore$ $E_y = 0 \Rightarrow V = ct$ if we move only in y-direction.

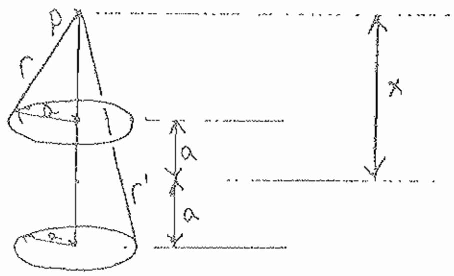
$\therefore$ The change in position in x-direction effects our answer.

$$|V_B - V_A| = \int_A^B \vec{E}.\vec{dx} = \int_A^B (\frac{\rho x}{\varepsilon o}).dx =$$

$$\frac{\rho}{\varepsilon o} . (\frac{x^2}{2})_{x_A}^{x_B} = \frac{\rho}{\varepsilon o} (\frac{d^2}{8} - 0) = \frac{\rho d^2}{8 \varepsilon o}$$

Problem 5:

Two identical rings of radius a carrying equal and opposite charges (uniformly distributed) q and -q are placed a distance 2a apart as shown in the figure.



(a) Derive the expression for the electric field $\vec{E}$ at point P. Deduce the expression of $\vec{E}$ for $x >> a$.

(b) Verify the results of part (a) by calculating the electric potential at P.

Solution:

(a) We will find the electric field at P due to the two rings separately, then we will superimpose them.

Upper ring: Because of symmetry only the vertical components of $\vec{E}$ will add up.

$$\vec{E}y = \vec{E} = \int d\vec{E} = \int dE \cos\theta$$

$$dE = \frac{1}{4\pi\varepsilon o} \frac{dq}{r^2}, \quad r^2 = a^2 + (x - a)^2$$

$$\cos\theta = \frac{(x-a)}{\left[(x-a)^2 + a^2\right]^{1/2}}$$

$$\therefore \quad E_1 = \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq\ (x-a)}{\left[(x-a)^2 + a^2\right]^{3/2}}$$

$$= \frac{(x-a)}{\left[(x-a)^2 + a^2\right]^{3/2} 4\pi\varepsilon_0} \cdot q$$

Lower ring:

Working similarly but realizing that $r'^2 = a^2 + (x + a)^2$, $\cos\theta = \frac{(x-a)}{r'}$ and the charge is negative, we get

$$E_2 = - \frac{(x + a)}{\left[(x+a)^2 + a^2\right]^{3/2} 4\pi\varepsilon_0} \cdot q$$

$$E = E_1 + E_2 = \frac{q}{4\pi\varepsilon_0} \cdot \left(\frac{(x-a)}{\left[(x-a)^2 + a^2\right]^{3/2}} - \frac{(x+a)}{\left[(x+a)^2 + a^2\right]^{3/2}} \right)$$

If $x >> a$

$$\frac{(x-a)}{\left[(x-a)^2 + a^2\right]^{3/2}} = \frac{(x-a)}{\left[x^2(1-\frac{a}{x})^2 + a^2\right]^{3/2}}$$

$$= \frac{x(1-\frac{a}{x})}{x^3 \left[(1-\frac{2a}{x}) + \frac{a^2}{x^2}\right]^{3/2}} = \frac{1}{x^2}\ (1 - \frac{a}{x}) \cdot (1 + \frac{3a}{x})$$

$$= \frac{1 + \frac{3a}{x} - \frac{a}{x} - \frac{3a}{x^2}}{x^2} = \frac{1 + \frac{2a}{x}}{x^2}$$

$$\frac{(x + a)}{\left[(x+a)^2 + a^2\right]^{3/2}} = \frac{(x + a)}{\left[x^2(1 + \frac{a}{x})^2 + a^2\right]^{3/2}}$$

$$= \frac{x(1 + \frac{a}{x})}{x^3 \cdot \left[(1+\frac{2a}{x}) + \frac{a^2}{x^2}\right]^{3/2}} = \frac{1}{X^2}\ (1 + \frac{a}{x}) \cdot (1 - \frac{3a}{x})$$

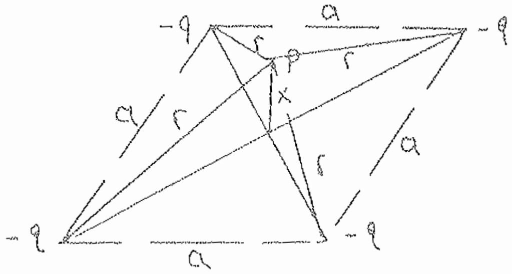
$$= \frac{1 - \frac{2a}{x} - \frac{3a^2}{x^2}}{x^2} = \frac{1 - \frac{2a}{x}}{x^2}$$

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0} \cdot \left(\frac{1 + \frac{2a}{x}}{x^2} - \frac{1 - \frac{2a}{x}}{x^2} \right)$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{4a}{x^3}$$

Problem 6:

Consider the charge configuration shown in the figure below.



(a) What is the electric field $\vec{E}$ at point P?

(b) What is the potential at point P? Can you verify this from your answer to a?

(c) What is the energy stored in this system? What does it mean physically ?

Solution:

(a) We will find the electric field of each charge at P and then will add them vectorially. Because of symmetry only the vertical components of electric field add up.

$$E_y = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot \cos\theta$$

$$r^2 = x^2 + \left(\frac{a}{\sqrt{2}}\right)^2 = x^2 + \frac{a^2}{2}$$

$$\cos\Theta = \frac{x}{r}$$

$$\therefore \quad E_y = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)x}{\left(x^2 + \frac{a^2}{2}\right)^{3/2}}$$

$$E_T = 4Ey = \frac{-4qx}{4\pi\varepsilon_0(x^2 + \frac{a^2}{2})^{3/2}}$$

(b) We will find the potential of each charge at P and then will add them algebraically.

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

$$V_T = 4V = -\frac{4}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + \frac{a^2}{2})^{1/2}}$$

$$V_T = -\int_{\infty}^{P} \bar{E}_T d\bar{\ell} = -\int E_T dx$$

$$= -\int -\frac{4qx}{4\pi\varepsilon_0(x^2 + \frac{a^2}{2})^{3/2}} \cdot dx$$

$$= \frac{4q}{4\pi\varepsilon_0} \int \frac{x}{(x^2 + \frac{a^2}{2})^{3/2}} \cdot dx \qquad U = x^2 - \frac{a^2}{2} \Rightarrow du = 2xdx$$

$$\therefore \quad V_T = \frac{4q}{4\pi\varepsilon_0} \int \frac{dU}{2U^{3/2}} = \frac{4q}{4\pi\varepsilon_0}\left(-\frac{2}{2} \cdot \frac{1}{U^{1/2}}\right)_{\infty}^{P}$$

$$= -\frac{4q}{4\pi\varepsilon_0} \cdot \frac{1}{U} = \frac{4q}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2 - \frac{a^2}{2})^{1/2}}$$

(c) Let U_T be the energy (potential) stored.

$$U_T = 6U = 6 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)(-q)}{a}$$

$$= \frac{6}{4\pi\varepsilon_0} \cdot \frac{q^2}{a}$$

The energy stored is positive, this means that we have to bring them together (positive work means force and distance have the same sense) and if we let free the system, they would do negative work to regain their zero potential

$\therefore$ Repulsion will take place.

Problem 7:

A nonconducting spherical shell of radius R has a charge of Q = 5c uniformly distributed on it. A charge q = -3c is placed at the center of the shell.

(a) Find the electric field outside the shell and inside the shell.

(b) Find the flux through a cube centered at the sphere and of side 5R. What is the flux through one surface?

(c) What is the answer to (b) if the side is only 1/2R?

Solution:

(a) Take the Gaussian surface as the sphere of radius r and center O.

$r > R \Rightarrow \oint \vec{E}.\vec{ds} = \oint E.ds = E \oint ds = E(4\pi r^2)$

But $\oint \vec{E}\vec{ds} = \frac{q_{net}}{\varepsilon_o}$

$$E(4\pi r^2) = \frac{(5-3)}{\varepsilon_o} = E = \frac{2}{4\pi r^2 \varepsilon_o}$$

$$= \frac{(10^9 \times 9)(2)}{r^2} = \frac{1.8 \times 10^{10}}{r^2}$$

$r < R$: $q_{net} = q = -3C$

$\therefore$ The electric field is directed towards the center.

$\oint \vec{E}.\vec{ds} = \oint -E.ds = -E(4\pi r^2)$

But $\oint \vec{E}.\vec{ds} = \frac{q_{net}}{o}$

$$\therefore \; -E(4\pi r^2) = \frac{-3}{\varepsilon_o} \Rightarrow E = \frac{3}{4\pi\varepsilon_o r^2}$$

$$E = \frac{(2.7 \times 10^{10})}{r^2}$$

(b) $\phi_T = \int \vec{E}\vec{ds} = \frac{q_{net}}{\varepsilon_o} = \frac{(5-3)}{\varepsilon_o} \quad \frac{2}{\varepsilon_o}$

$$= \frac{2}{8.85 \times 10^{-12}} = 2.26 \times 10^{11} \text{ Wb}$$

$$\phi_S = \frac{\phi_T}{6} = \frac{2.26 \times 10^{11}}{6} = 3.76 \times 10^{10} \text{ Wb}$$

(c) $\phi_T = \int \vec{E}.\vec{ds} = \frac{q_{net}}{\varepsilon_o} = \frac{-3}{\varepsilon_o} = \frac{-3}{8.85 \times 10^{-12}}$

$$= -3.38 \times 10^{11} \text{ Wb}$$

$$\phi_S = \frac{\phi_T}{6} = -\frac{3.38 \times 10^{11}}{6} = 5.64 \times 10^{10} \text{ Wb}$$

Problem 8:

A rod of length L = 2m has a uniform charge density = 3 c/m.

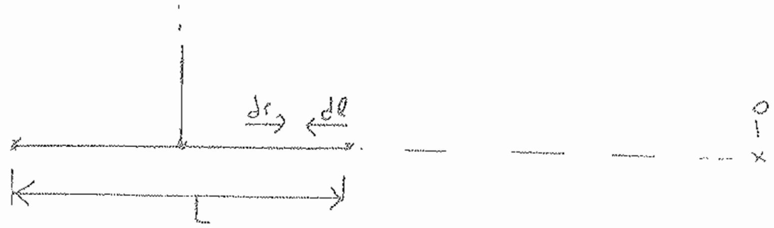
(a) Find the potential at a point P a distance 5m from the center of the rod and alongits axis.

(b) Find the electric field by direct integration at that point.

(c) How are the answers of (b) and (a) related? Explain.

Solution:

(a) $dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r}$



$dq = \lambda\, d\ell$

Note: We can take $d\ell$ pointing to right or to left. The final answers will remain the same. Here, we take $d\ell$ to the left as shown in the figure.

$dr = d\ell$ (If ℓ increases then r increases)

r extends from (5-1=)4m to (5+1=)6m.

$$V = \int dV = \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r} = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int_4^6 \frac{dr}{r} = \frac{\lambda}{4\pi\varepsilon_0} \ln(3/2)$$

$$= (9 \times 10^9)(3)(\ln(3/2)) = 1.09 \times 10^{10} V$$

(b) $$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} \Rightarrow E = \int dE = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int \frac{dr}{r^2} = \frac{\lambda}{4\pi\varepsilon_0} \left(-\frac{1}{r}\right)_4^6$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left(-\frac{1}{6} + \frac{1}{4}\right) = (9 \times 10^9)\cdot(3)\cdot(0.0833)$$

$$= 2.25 \times 10^9 \text{ V/m}$$

(c) For a more general case where P can move in the x direction.

$$V = \int dV = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int_{(r-1)}^{(r+1)} \frac{dr}{r} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{r+1}{r-1}\right)$$

$$\frac{dV}{dr} = -\frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{2}{(r-1)(r+1)}.$$

$$\therefore\ -\frac{dV}{dr} = -\frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{2}{(r-1)(r+1)}$$

$$E = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int \frac{dr}{r^2} = \frac{\lambda}{4\pi\varepsilon_0} \cdot \left(-\frac{1}{r}\right)_{r-1}^{r+1}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left(-\frac{1}{r+1} + \frac{1}{r-1}\right) = \frac{\lambda}{4\pi\varepsilon_0} \left(\frac{-r+1+r+1}{(r+1)(r-1)}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{2}{(r-1)(r+1)}$$

As we proved $E = -\frac{dV}{dr}$

Extra Problem:

Two nonconducting spherical shells of equal radii (R = 3cm) and having charges of 1C and 3C. In both cases the charge is uniformly distributed over the shells. The centers of the shells are separated by a distance of 15cm.

(a) Sketch the form of the electric potential along a line joining their centers indicating the positions of maximum and minimum potential.

(b) What is the value of the electric field at these points?

CAPACITORS & DIELECTRICS

Problem 1 :

A capacitor C_1 is charged to a voltage V. It is then connected to a second capacitor C_2 which is initially uncharged .

a) Calculate the final charge on C_2 in terms of C_1, C_2 and V.

b) Calculate the change in energy of the system in terms of C_1, C_2 and V .

Solution:

(a) When, at first C_1 is connected to a voltage the charge is $q_0 = C_1 V$.

Conservation of charge $\Longrightarrow$ $q_0 = q_1 + q_2$

$$V_{ab} = \frac{q_1}{C_1} = \frac{q_2}{C_2} \Longrightarrow q_1 = \frac{C_1 q_2}{C_2}$$

Substituting for $q_0 = q_1 + q_2$ we get

$$C_1 V = \frac{C_1 q_2}{C_2} + q_2 = q_2 \left(\frac{C_1}{C_2} + 1 \right)$$

$$\therefore q_2 = \frac{C_1 V}{\left({}^{C_1}/{}_{C_2} + 1 \right)} \quad \text{(coulomb)}$$

(b) $U_I = \frac{1}{2} C_1 V^2$

$$U_F = \tfrac{1}{2} C_1 V_{ab}^2 + \tfrac{1}{2} C_2 V_{ab}^2 = \frac{V_{ab}^2}{2} (C_1 + C_2)$$

But $q_0 = q_1 + q_2 \Longrightarrow C_1 V = C_1 V_{ab} + C_2 V_{ab} \Longrightarrow$

$$V_{ab} = \frac{C_1 V}{C_1 + C_2}$$

$$\therefore U_F = \tfrac{1}{2} \cdot \left(\frac{C_1 V}{C_1 + C_2} \right)^2 \cdot (C_1 + C_2) = \frac{C_1^2 V^2}{2(C_1 + C_2)}$$

$$\therefore W = U_F - U_I = \tfrac{1}{2} C_1 V^2 \left(\frac{C_1}{C_1 + C_2} \right) - \tfrac{1}{2} C_1 V^2$$

$$= \frac{C_1 V^2}{2} \cdot \left(\frac{C_1 - C_1 - C_2}{C_1 + C_2} \right) = - \frac{C_1 V^2}{2} \left(\frac{C_2}{C_1 + C_2} \right) .$$

Problem 2 :

(a) An unknown capacitance C_x is connected in parallel to a capacitor of capacitance C = 100 m.f. originally charged to a voltage V.
If the voltage drops to V/3 what is the capacitance of the unknown capacitance C_x.

(b) If we had connected the capacitors in series what would the potential across both of them be .

Solution:

(a) Because of conservation of charge

$$q = q_1 + q_2$$

$$\therefore CV = CV_{ab} + C_x V_{ab}$$

$$\therefore CV = \frac{V}{3}(C + C_x)$$

$$\therefore 2C = C_x$$

$$\therefore C_x = (2)(0.1) = 0.2\ F = 200\ mF.$$

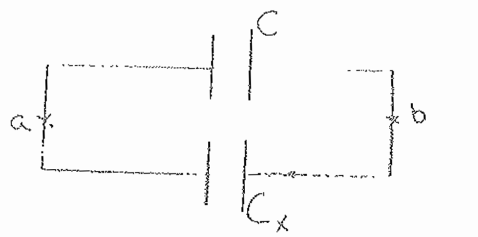
(b) $$\frac{1}{Ceq} = \frac{1}{C} + \frac{1}{C\ x} = \frac{1}{100} + \frac{1}{200} = \frac{3}{200}$$

$$\therefore Ceq = \frac{200}{3}\ mF\ .$$

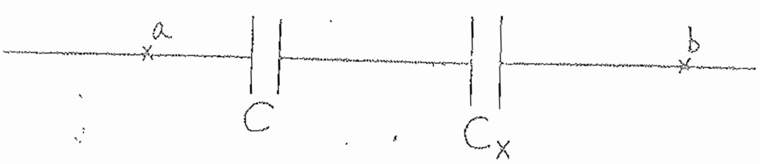
$$q = CV_I = 100\ V_I$$

$$V = \frac{q}{Ceq} = \frac{(100\ V_I)(3)}{200} = 1.5\ V_I$$

(a)



(b)



Problem 3 :

A parallel plate capacitor (C_1 = 100 microfarads) is charged to a potential V = 10 V. The charging battery is then removed and the plates are brought closer together until their separation is one half of its original value .

a) What is the potential difference between the plates now ? (Explain clearly). What is the charge on the plates ?

b) If the space between the plates of the capacitor is then filled with material of dielectric constant x = 1.5 what is the difference of potential between the plates now ?

Solution:

(a) Initially the charge $q = C_1V = (10^{-4})(10) = 1 \times 10^{-3}$ C

But when the battery is removed the voltage could be changed but the charge remains the same (conservation of charge) therefore, the charge on the plates is 1×10^{-3} C .

$$\left.\begin{array}{l} C_1 = \dfrac{\varepsilon_\circ A}{d_1} \\ C_2 = \dfrac{\varepsilon_\circ A}{d_2} \end{array}\right] \Longrightarrow \frac{C_1}{C_2} = \frac{d_2}{d_1} = \tfrac{1}{2}$$

But as we notice q is conserved .

$$\therefore q = C_1V_1 = C_2V_2$$

$$\therefore \tfrac{1}{2} = \frac{C_1}{C_2} = \frac{V_2}{V_1}$$

$$\therefore \frac{V_2}{V_1} = \tfrac{1}{2} \Longrightarrow V_2 = \frac{10}{2} = 5V .$$

(b) $\dfrac{V_o}{V} = x \Longrightarrow V = \dfrac{V_o}{x} = \dfrac{5}{1.5} = 3.33$ V

Problem 4:

A parallel plate capacitor is charged to a potential V_1. The charging battery is then removed and the plates are pulled apart until their sep ration is doubled .

a. What is the potential difference between the plates now ?

b. If the space between the plates is then filled with material of dielectric constant K .
What is the potential difference across the plates of the capacitor now ?

Solution:

(a)
$$\left.\begin{aligned} C_1 &= \frac{\varepsilon_0 A}{d_1} \\ C_2 &= \frac{\varepsilon_0 A}{d_2} \end{aligned}\right] \Longrightarrow \frac{C_1}{C_2} = \frac{d_2}{d_1} = 2$$

But, because of conservation of charge

$$q = C_1 V_1 = C_2 V_2$$

$$\therefore \frac{C_1}{C_2} = \frac{V_2}{V_1} = \frac{d_2}{d_1} = 2$$

$$\therefore V_2 = 2V_1 .$$

(b) $$\frac{V_2}{V} = K \Longrightarrow V = \frac{V_2}{K} = \frac{(2V_1)}{K} = \frac{2V_1}{K} \text{ volts.}$$

Problem 5:

(a) Consider a parallel plate capacitor of area $A = 100\ cm^2$ and separation $d = 1$ mm. If the space between the plates is filled with dielectric of constant $K = 3$,
The capacitance of this capacitor is ...?

(b) If the capacitor is charged to a potential $V = 100$ volts and then disconnected from the charging battery. Suppose now we, remove the piece of dielectric, the potential now across the plates is ...?

(c) A parallel plate capacitor of capacitance 100 uF is now filled half way with dielectric of constant $K = 3$. The new capacitance of the capacitor is :

AUBT C = 200 uF

JVUV C = 100 uF

LXMP C = 150 uF

NYTX C = 66.66 uF

None of the above my answer is : ______________

Solution:

(a) For the parallel plate C is given as

$$C = \frac{\epsilon_0 A}{d} \cdot K = \frac{(8.85 \times 10^{-12}).(10^{-2})(3)}{(10^{-3})}$$

$= 2.65 \times 10^{-12}$ F.

(b) Initially $q_I = C_I V_I = 100\ C_d$.

Finally $C_F = \frac{C_d}{K} = \frac{C_d}{3}$

and $q_I = C_F V_F = \frac{C_d V_F}{3}$

but ; because of conservation of charge $q_I = q_F$

$\therefore 100\ C_d = \frac{C_d V_F}{3}$

or $V_F = 300$ V .

(c) This combination can be considered as two capacitors in parallel .

$C_1 = \frac{100}{2}\ \mu F = 50\ \mu F$ (Half the area of the capacitor).

$C_2 = (3)\ (\frac{100}{2}) = 150\ \mu F$ (second half of the area filled with a dielectric of K = 3).

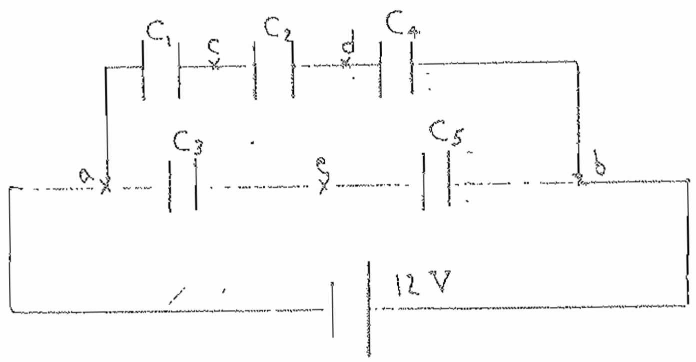
$Ceq. = C_1 + C_2 = 50 + 150 = 200\ \mu F$.

Problem 6:

Consider the capacitors arranged as in the following figure with $C_1 = 1$ uF, $C_2 = 2$ uF, $C_3 = 3$ uF, $C_4 = 4$ uF, and $C_5 = 5$ uF. If the potential across the terminals of the battery is 12 V .

a. What is the net capacitance of these five capacitors as arranged ?

b. What is the charge on each capacitor ?



Solution:

(a) Let $C_{1,2,4} = C_a$

$C_{3,5} = C_b$

$C_{1,2,3,4,5} = C$,

$$\frac{1}{C_a} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_4} = \frac{C_1C_2 + C_1C_4 + C_2C_4}{C_1C_2C_4}$$

$$\therefore C_a = \frac{C_1C_2C_4}{C_1C_2 + C_1C_4 + C_2C_4} = \frac{(1)(2)(4)}{(1)(2)+(1)(4)+(2)(4)}$$

$$= 0.571 \text{ uF} .$$

$$\frac{1}{C_b} = \frac{1}{C_3} + \frac{1}{C_5} = \frac{C_3 + C_5}{C_3C_5}$$

$$\therefore C_b = \frac{C_3C_5}{C_3 + C_5} = \frac{(3)(5)}{(3+5)} = 1.875 \text{ uF} .$$

$$C = C_a + C_b = 0.571 + 1.875 = 2.446 \text{ uF} .$$

(b) From the conservation of charge

$q_b = q_3 = q_5$ & $q_a = q_1 = q_2 = q_4$.

$$q = CV = (2.446 \times 10^{-6}).(12) = 2.93 \times 10^{-5} \text{ C}$$

$$Vab = \frac{q_a}{C_b} = \frac{q_b}{C_b} \Longrightarrow C_aq_b = C_bq_a \Longrightarrow$$

$$(0.571)q_b = (1.875)q_a \Longrightarrow q_b = (3.284)q_a$$

but $q = 2.93 \times 10^{-5} = q_a + q_b$

Solving the two equations we get

$$q_a = q_1 = q_2 = q_4 = 6.84 \times 10^{-6} \text{ C}$$

$$q_b = q_3 = q_5 = 2.246 \times 10^{-5} \text{ C}.$$

Problem 7:

An electron is located between two conducting parallel plates. One plate is at a potential of +25v and the other plate is at a potential of −5v. The two plates are separated by a distance of 1.5 cm.

a. If the electron is 0.5 cm away from the plate with a negative potential, what is the force which the electron experiences

$e = 1.6 \times 10^{-19}$ C $\varepsilon_0 = 8.9 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

b. What is the difference in the energy of the electron if it is moved from the point above to a point 0.2 cm away from the plate with positive potential.

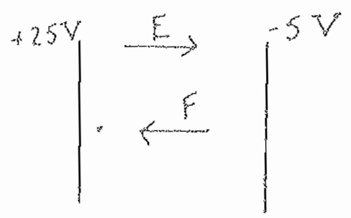
Solution:

(a) In two conducting parallel plates the electric field is uniform & so

$$V_{ab} = Ed \Longrightarrow E = \frac{V_{ab}}{d} = \frac{25 - (-5)}{1.5 \times 10^{-2}} = 0.2 \times 10^{+4}\ \text{V/m}$$

The force on the electron is given by :

$$|F| = q.E = (1.6 \times 10^{-19}).(2 \times 10^{3}) = 3.2 \times 10^{-16}\ \text{N}.$$



(b) The distance traveled is : $x_F - x_I = 1 - 0.2 = 0.8$ cm

The difference in energy is the work done when the electron is moved from the initial to the final position.

$$|\Delta E| = |W| = \int_{x_I}^{x_F} F.dx = (3.2 \times 10^{-16}).(x_F - x_I)$$

$$= (3.2 \times 10^{-16}).(0.8 \times 10^{-2}) = 2.56 \times 10^{-18}\ \text{J}.$$

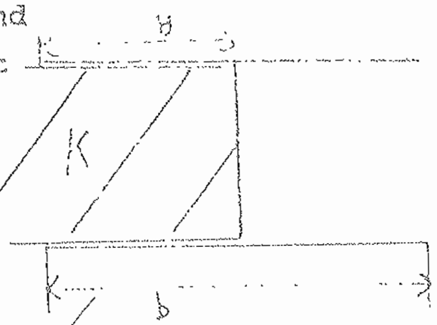
Problem 8 :

An isolated parallel plate capacitor of sides a and b and plate separation x carries a charge Q.

a) What is in terms of the data of the problem the energy stored in the capacitor ? What is the new energy if the distance is increased by dx ?

b) When the plate separation is x, an external agent exerts a force F on one of the plates to increase the distance between the plates by dx .
What is the force F ? What is the force of attraction between the plates ?

c) A dielectric of dielectric constant K = 2 is introduced in the capacitor and fills it to a distance y. What is the energy stored in the capacitor ? Calculate the change in energy dU if y is changed by dy. What is therefore the force of attraction between the capacitor and the dielectric ?



Solution:

(a) $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 (ab)}{x}$

$$U_o = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2} \cdot \frac{x}{\varepsilon_0 (ab)}$$

$$dU = \frac{Q^2}{2} \cdot \frac{dx}{\varepsilon_0 (ab)}$$

∴ If the distance is increased by dx the energy is increased by dU and the final energy becomes :

$$U = U_o + du = \frac{Q^2}{2}\frac{(x + dx)}{\varepsilon_0 (ab)}$$

(b) $dU = dw = Fdx = \frac{Q^2}{2} \cdot \frac{dx}{\varepsilon_0 (ab)}$

$\therefore F = \frac{Q^2}{2\varepsilon_0 \; ab}$ (away from the plate)

The work is done in equilibrium states, therefore the force of attraction is equal to the force needed to increase the distance between the plates.

$\therefore F = \frac{Q^2}{2\varepsilon_0 ab}$ (pointing to the other plate).

(c) This arrangement is equivalent of having two variable capacitors (if y changes) put is parallel, the first one (C_1) without the dielectric while the second (C_2) with the dielectric).

$$C_1 = \frac{\varepsilon_0 (a\,(b-y))}{x} \quad C_2 = \frac{k\varepsilon_0 (ay)}{x}$$

$$C = C_1 + C_2 = \frac{\varepsilon_0 a}{x} \cdot \Big((b-y) + Ky \Big) = \frac{\varepsilon_0 a\,(b+y\,(K-1))}{x}$$

$$U = \tfrac{1}{2} \frac{Q^2}{C} = \tfrac{1}{2} Q^2 \cdot \left(\frac{x}{\varepsilon_0 \; a\,(b+y\,(K-1))} \right)$$

$$= \frac{Q^2 \, x}{2\,\varepsilon_0 \, a \, [(b+y(K-1)]}$$

$$\frac{dU}{dy} = \frac{-(K-1)\; Q^2 \; x}{2\,\varepsilon_0\, a \cdot [b+y\,(K-1)]^2}$$

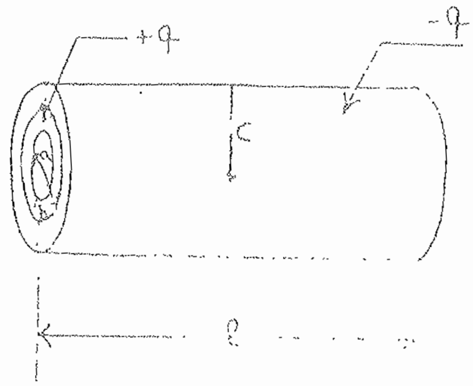
$$\therefore dU = \frac{-(K-1)\; Q^2 \; x \; dy}{2\,\varepsilon_0\, a\, [b+y(K-1)]^2}$$

But $dU = dW = Fdy$

$$\therefore F = \frac{-(K-1)\; Q^2 \; x}{2\,\varepsilon_0\, a\, [b+y(K-1)]^2}$$

Problem 9:

Consider two concentric conducting cylinders of radii a and c (c > a) and length ($\ell \gg c$). The region between these cylinders is filled between r = a and r = b (b < c) with dielectric material of dielectric constant K. The region between r = b and r = c is empty. There is a charge +q on the inner conducting cylinder and -q on the outer conducting cylinder.



a. What are the values of D, E and P every where in space (consider each region separately).

b. What is the potential difference between the inner conducting cylinder and the outer conducting cylinder.

c. What is the capacitance of the system.

Solution:

(a) A convenient Gauss's surface is a circular cylinder of radius r and length l closed at each end by plane caps normal to the axis. There is no flux through the circular caps because $\vec{E}$ lies in the surface at every point.

$r < a$: $\oint \vec{E} \cdot \vec{ds} = E \oint ds = E\,(2\pi r l) = \frac{q_{net}}{\varepsilon_\circ}$

$q_{net} = 0 \Longrightarrow E = 0$

But $D = K \cdot \varepsilon_\circ \cdot E$ & $P = \varepsilon_\circ (K-1) E$

$\therefore D = 0$ & $P = 0$.

$a < r < b$: $\oint K\vec{E} \cdot \vec{ds} = K \oint E . ds = KE \oint ds$

$= KE(2\pi r l) = \frac{q_{net}}{\varepsilon_\circ}$

$\therefore E = \frac{q}{\varepsilon_\circ K 2\pi \cdot r l}$

$D = K \cdot \varepsilon_\circ \cdot E = \frac{q}{2 \cdot \pi r l}$

$P = \varepsilon_\circ (K-1) E = \frac{q(K-1)}{2 \cdot \pi r l K}$

$b < r < c$: $\oint \vec{E} \cdot \vec{ds} = E(2 \cdot \pi r l) = \frac{q_{net}}{\varepsilon_\circ}$

$\therefore E = \frac{q}{2 \cdot \pi r l \varepsilon_\circ}$

$D = \varepsilon_\circ E = \frac{q}{2\pi r l}$

$P = \varepsilon_\circ (K-1)\, E = \varepsilon_\circ (1-1)\, E = 0$.

$r > c$: $\oint \vec{E} \cdot \vec{ds} = E \cdot (2\pi r l) = \frac{q_{net}}{\varepsilon_\circ}$

But $q_{net} = q - q = 0$

$\therefore E = 0$

Therefore $D = 0$ & $P = 0$

(b) $V = -\int_a^c \vec{E} \cdot \vec{dl} = -\int_a^b \vec{E} \cdot \vec{dl} - \int_b^c \vec{E} \cdot \vec{dl}$

Notice: $\vec{dl} = \vec{dr}$ & $\vec{dE} \cdot \vec{dl} = dE . dl . \cos 180^\circ = -dE . dl$

$\therefore V = + \int_a^b E dr + \int_b^c E dr$

$$= \int_a^b \frac{q \cdot dr}{(\varepsilon_\circ K 2\pi \cdot l)\, r} + \int_b^c \frac{q}{(\varepsilon_\circ 2\pi \cdot l) r} \cdot dr$$

$$= \frac{q}{K \cdot \varepsilon_0 2\pi \cdot l} \cdot \left[\ln(b/a) + K\ln(c/b) \right]$$

$$= \frac{q}{K\varepsilon_0 \cdot 2\pi \cdot l} \cdot \ln\left(\frac{c^K}{ab^{K-1}} \right)$$

(c) $C = \frac{q}{V} = \frac{K\varepsilon_0 \cdot 2\pi \cdot l}{\ln\left(\frac{c^K}{ab^{K-1}}\right)}$

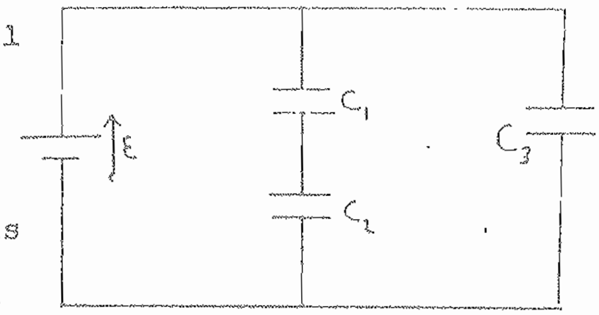
Problem 10:

Consider the following circuit with $C_1 = 10$ uF , $C_2 = 5$ uF , $C_3 = 4$ uF and $\varepsilon = 100$ V.

a. Find the charge for each capacitor.

b. Find the potential difference across each capacitor .

c. If capacitor C_3 is now filled with a dielectric of dielectric constant K = 2, how do your answers to (a) and (b) above change if at all?



Solution:

(a) $C_{1,2} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(10)(5)}{(10+5)} = 3.33$ uF .

$C = C_{1,2} + C_3 = 3.33 + 4 = 7.33$ uF .

$q = C\varepsilon = (7.33 \times 10^{-6}).(100) = 7.33 \times 10^{-4}$ C.

$q_{1,2} = q_1 = q_2$ & $q = q_{1,2} + q_3 = 7.33 \times 10^{-4}$

$\frac{q_{1,2}}{C_{1,2}} = \frac{q_3}{C_3} \Longrightarrow 1.2\, q_{1,2} = q_3$

Solving the two equations

$q_{1,2} = q_1 = q_2 = 3.33 \times 10^{-4}$ C

$q_3 = 4 \times 10^{-4}$ C

(b) $V_1 = \frac{q_1}{C_1} = \frac{3.33 \times 10^{-4}}{10 \times 10^{-6}} = 33.33$ V.

$V_2 = \frac{q_2}{C_2} = \frac{3.33 \times 10^{-4}}{5 \times 10^{-6}} = 66.66$ V .

$V_3 = \mathcal{E} = 100$ V .

(c) $C_3' = 2C_3 = 8$ uF .

$C = C_{1,2} + C_3' = 3.33 + 8 = 11.33$ uF .

$q = C\mathcal{E} = (11.33 \times 10^{-6})(100) = 11.33 \times 10^{-4}$ C

$\frac{q_{1,2}}{C_{1,2}} = \frac{q_3}{C_3'} \Longrightarrow 2.4\, q_{1,2} = q_3$

But $q = 11.33 \times 10^{-4} = q_{1,2} + q_3$

Solving the two equations :

$q_{1,2} = q_1 = q_2 = 3.33 \times 10^{-4}$ C

$q_3 = 8 \times 10^{-4}$

We notice that q_1 & q_2 didn't change . This result is logical because the branch of C_1 & C_2 is independent of the branch of C_3 & there is still a potential of 100 V across C_1 & C_2 as before . As for C_3, q is just doubled because K = 2 .

V_1 & V_2 will not change because q_1, q_2, C_1 & C_2 remained the same .

$V_3 = \mathcal{E} = 100$ V no matter what we do with dielectries .

$q_3 = C_3V$ where V is constant if we increase C by putting a dielectric, q increases but V remains constant . Bearing these concepts in mind we could solve the last part without repeating again all the calculations .

CURRENT & RESISTANCE

Problem 1 :

An aluminum wire ($\rho_a = 2.8 \times 10^{-8}$ ohm-m) and a tungsten wire ($\rho_t = 5.6 \times 10^{-8}$ ohm-m) of the same radius have ths same potential difference applied to them. What must be the ratio of their lengths

1) if the current is to be the same in the two wires.
2) if the electric field is to be the same in the two wires .

What is in each case the ratio of the resistance $\frac{R_a}{R_t}$?

Solution :

(1) $I_a = I_t$, $V_a = V_t$, $V_a = I_a R_a$ &
$V_t = I_t R_t$ Besides if $r_a = r_t \Longrightarrow A_a = A_t$

$$\therefore R_a = R_t \Longrightarrow P_a \frac{l_a}{A} = P_t \frac{l_t}{A}$$

$$\text{or } \frac{\rho_t}{\rho_a} = \frac{l_a}{l_t} = \frac{5.6 \times 10^{-8}}{2.8 \times 10^{-8}} = 2$$

$$\therefore \frac{l_a}{l_t} = 2 .$$

(2) The conductor is a long cylinder of a fixed cross section A , therefore the electric field does not change from place to place in each conductor .

$$V = - \int \vec{E} \cdot \vec{dl} = El \quad \text{or } l = \frac{V}{E}$$

But both V & $\vec{E}$ have the same value i.e. $V_a = V_t$ & $E_a = E_t$.

$$\therefore l_a = l_t \Longrightarrow \frac{l_a}{l_t} = 1$$

For (2) $\rho_a = \frac{E}{ja}$ & $\rho_t = \frac{E}{jt}$

$\therefore \rho_a i_a = \rho_t . j_t \Longrightarrow \rho_a j_a A = \rho_t j_t A \Longrightarrow$

$\rho_a i_a = \rho_t i_t \Longrightarrow \frac{i_a}{i_t} = 2$

$\frac{R_a}{R_t} = (\frac{V}{i_a}).(\frac{i_t}{V}) = ½$

For (1) $R_a = R_t \Longrightarrow \frac{R_a}{R_t} = 1$.

Problem 2:

A copper wire of length l and diameter d is joined to a nickel wire of the same length and diameter .

The resistivity of nickel is four times as big as the resistivity of copper. A difference of potential of 100 volts is applied between the ends of the whole combination of wires .

a. What is the difference of potential across the copper wire .

b. What is the ratio $\frac{E_{cu}}{E_{Ni}}$ of the electric fields in the two wires .

c. Find the ratio $\frac{P_{cu}}{P_{Ni}}$ of the rate of development of heat in the two wires .

Solution:

(a) $d_c = d_n \Longrightarrow \quad A_c = A_n = A$

$$R_c = \frac{\;_c l}{A} = (\frac{4\;\;_c l}{A}).(\frac{1}{4}) = (\frac{\;\;_n l}{A}).(\frac{1}{4}) = \frac{R_n}{4}$$

$\therefore \; 4R_c = R_n$

$$V_c = \frac{V \; R_c}{R_n + R_c} = \frac{V \; R_c}{5R_c} = \frac{V}{5} = \frac{100}{5} = 20 \text{ V.}$$

$$V_n = V - V_c = 80 \text{ V}$$

(b) The conductors are long cylinder of fixed cross section A . Therefore

$$V = - \int \vec{E} \, \vec{dl} = El \Longrightarrow E = V/l$$

$$\frac{E_c}{E_n} = (\frac{V_c}{l}).(\frac{l}{V_n}) = (\frac{V_c}{l})(\frac{l}{4V_c}) = \frac{1}{4}$$

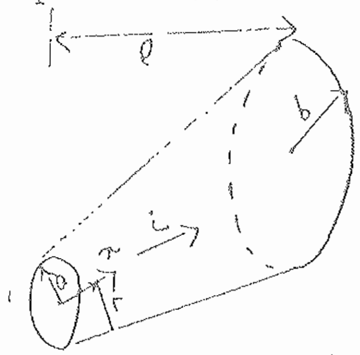
(c) $\frac{P_c}{P_n} = \frac{V_c I}{V_n I} = \frac{V_c}{4V_c} = \frac{1}{4}$.

Problem 3:

A resistor is in the shape of a truncated right circular cone . The end radii are a and b , the altitude is l. If the taper is small, we may assume that the current density is uniform across any cross section .

a. Calculate the resistance of this object.

b. Show that your answer reduces to $\rho(l/A)$ for the special case of zero taper (a = b).



Solution:

(a) $dR = \rho \frac{dx}{A} \quad 0 < x < l$

$A = \pi r^2$ angle of taper $= \alpha$

$\tan\alpha = \frac{(b - a)}{l}$

$\tan\alpha = \frac{(r - a)}{x} \Longrightarrow dx = \frac{dr}{\tan\alpha} = \frac{l\,dr}{(b - a)}$

$$R = \int dR = \int \rho \frac{dx}{A} = \int \rho \frac{dx}{\pi \cdot r^2} = \frac{\rho}{\pi} \cdot \int_a^b \left(\frac{l\,dr}{(b-a)}\right) \cdot \frac{1}{r^2}$$

$$= \frac{\rho l}{\pi (b-a)} \left(-\frac{1}{r}\right)_a^b = \frac{\rho l}{\pi (b-a)} \cdot \left(\frac{b-a}{ab}\right) = \frac{\rho l}{\pi ab}$$

(b) Zero taper $\Longrightarrow \tan\alpha = 0 \Longrightarrow b - a = 0 \Longrightarrow b = a$

$$\therefore R = \frac{\rho l}{\pi (a)(a)} = \frac{\rho l}{\pi \cdot a^2} = \rho \frac{l}{A} .$$

Problem 4:

A 1250-W radiant heater is constructed to operate at 115 V.

a. What will be the current in the heater ?

b. What is the resistance of the heating coil ?

c. How many kilocalories are generated in one hour by the heater ?

Solution:

(a) $P = VI \Longrightarrow I = \frac{P}{V} = \frac{1250}{115} = 10.87 \text{ A} \approx 11 \text{ A}$

(b) $P = \frac{V^2}{R} \Longrightarrow R = \frac{V^2}{P} = \frac{(115)^2}{1250} = 10.6 \ \Omega$

(c) $W = P.t = (1250).(3600) = 4500 \text{ KJ}$

$= \frac{4500}{4.185} \text{ Kcal} = 1075 \text{ Kcal} \simeq 1100 \text{ Kcal}$.

ELECTROMOTIVE FORCE AND CIRCUITS

Problem 1:

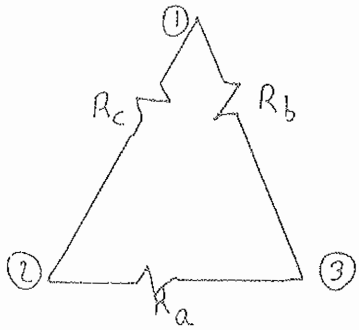
a. Three resistances R_a, R_b and R_c are connected as shown. We measure the resistances between the terminals 1, 2, 3. The resistances measured are :

$R_{12} = 5\,\Omega$

$R_{23} = 10\,\Omega$

$R_{31} = 11\,\Omega$

Find the resistances R_a, R_b and R_c.



b. If a battery of EMF $\mathcal{E} = 12V$ is connected between the terminals 2 and 3 find the potential difference between points 1 and 2.

Solution:

(a) $$\frac{1}{R_{12}} = \frac{1}{R_c} + \frac{1}{R_b + R_a} \Longrightarrow R_{12} = \frac{R_c(R_b + R_a)}{R_b + R_a + R_c} = 5$$

$$\frac{1}{R_{23}} = \frac{1}{R_a} + \frac{1}{R_b + R_c} \Longrightarrow R_{23} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = 10$$

$$\frac{1}{R_{31}} = \frac{1}{R_b} + \frac{1}{R_a + R_c} \Longrightarrow R_{31} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} = 11$$

Solving for the three equations and three unknowns we get $R_c = 5.75\,\Omega$, $R_a = 23\,\Omega$ & $R_b = 15.33\,\Omega$.

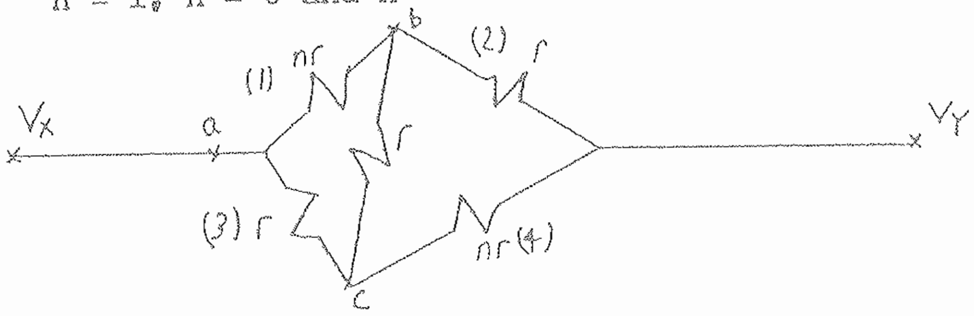
(b) $$i_c = i_b = \frac{\mathcal{E}}{R_b + R_c} = \frac{12}{21.083} = 0.57\text{ A}$$

$$V_{12} = i_c R_c = (0.57).(5.75) = 3.27\text{ V}$$

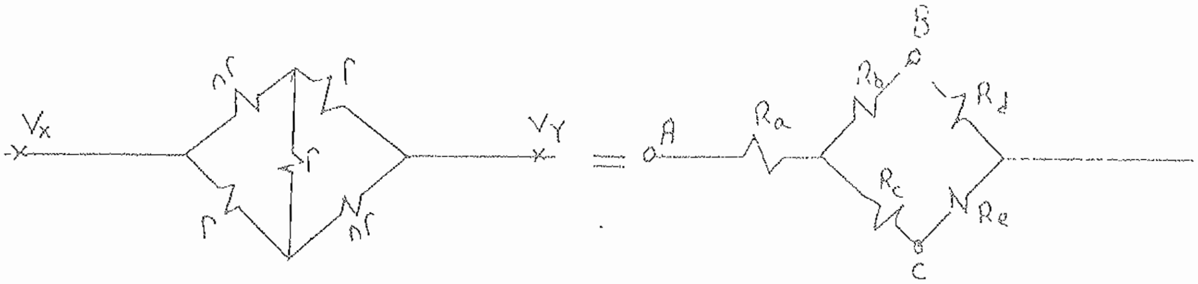
Problem 2 :

What is the equivalent resistance between the terminals x and y for the circuit shown below . Verify your answer for the special cases where

$n = 1$, $n = 0$ and $n = \infty$



Solution :



This conversion is from delta (Δ) to wye (Y), we can find the resistances of the wye type , i.e. R_a, R_b and R_c by the converting formulaes :

$$R_a = \frac{R_{ab}\ R_{ac}}{R_{ab} + R_{ac} + R_{bc}} = \frac{(nr)(r)}{nr + r + r} = \frac{nr}{(2 + n)}$$

$$R_b = \frac{R_{ba}\ R_{bc}}{R_{ab} + R_{ac} + R_{bc}} = \frac{(nr)(r)}{nr + r + r} = \frac{nr}{(2 + n)}$$

$$R_c = \frac{R_{ca}\ R_{cb}}{R_{ab} + R_{ac} + R_{bc}} = \frac{(r)(r)}{nr + r + r} = \frac{r}{(2 + n)}$$

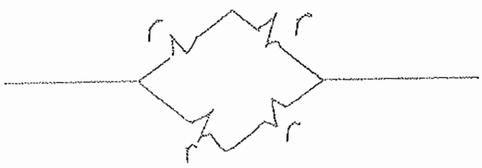
$$\therefore \text{Req} = R_a + \frac{(R_b + R_d)\cdot(R_c + R_e)}{R_b + R_d + R_c + R_e}$$

$$= \frac{nr}{(2+n)} + \frac{(\frac{nr}{2+n} + r)(\frac{r}{2+n} + nr)}{(\frac{nr}{2+n}) + r + (\frac{r}{2+n}) + nr}$$

$$= \frac{nr}{(2+n)} + \frac{2r(n+1)^2}{(n+3)(n+2)}$$

$$= \frac{r(n+2)(3n+1)}{(n+3)(n+2)} = \frac{r(3n+1)}{(n+3)}$$

For n = 1, because of symmetry no current passes through r_{BC} (in the middle) so the system reduces to :

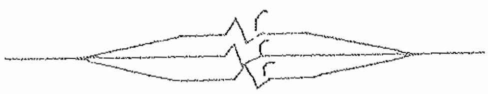


where $Req = \frac{(2r)(2r)}{2r + 2r} = r$

Similarly from the equation .

$$Req = \frac{r(4)}{4} = r \qquad (n = 1)$$

For n = 0 the system reduces to :

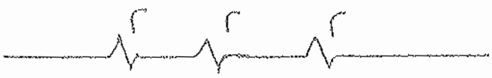


where Req = r/3

Similarly from the equation

$$Req = \frac{r(3n+1)}{(3+n)} = \frac{r}{3} \qquad (n = 0)$$

For $n = \infty$ the system reduces to :



where Req = 3r

Similarly from the equation

$$Req = \frac{r(3n+1)}{(n+3)} = \frac{r(3+1/n)}{1+3/n}$$

as $n \longrightarrow \infty$ $Req = \frac{3r}{1} = 3r$.

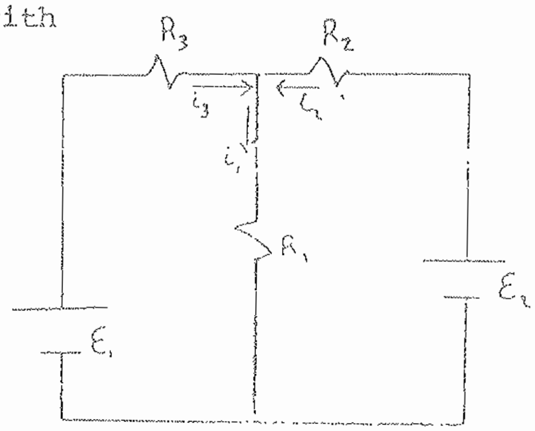
Problem 3:

Consider the circuit shown with

$\mathcal{E}_1 = 3$ V. $\mathcal{E}_2 = 6$ V.

$R_1 = 5\Omega$, $R_2 = 4\Omega$ and

$R_3 = 8\Omega$



a. Find the currents i_1 , i_2 , i_3 in the resistors R_1, R_2 & R_3 respectively.

b. What power appears as thermal energy in each of the three resistors ?

c. What power is supplied by the batteries, is energy conserved ?

Solution:

(a) Writing K.V.L. (Kirchhoff's Voltage Law) to the two "windows" we get :

$$\mathcal{E}_1 = i_3R_3 + i_1R_1 \quad --- \quad (1) \quad ---$$

$$\mathcal{E}_2 = i_2R_2 + i_1R_1 \quad --- \quad (2) \quad ---$$

Besides, using K.C.L. (Kirchhoff's Current Law) to the node at A :

$$i_3 + i_2 = i_1. \quad --- \quad (3) \quad ---$$

Substituting : (1), (2) & (3)

$$3 = 8i_3 + 5i_1 = 8i_3 + 5(i_3 + i_2) = 13i_3 + 5i_2$$

$$6 = 4i_2 + 5i_1 = 4i_2 + 5(i_3 + i_2) = 9i_2 + 5i_3$$

Solving for i_2 & i_3 , we get : $i_3 = -0.033$ A and $i_2 = 0.685$ A .

i_3 is found negative indicating that the direction should be opposite to that shown in figure .

& $i_1 = i_2 + i_3 = 0.652$ A

(b) $P_1 = i_1{}^2R_1 = (0.652)^2.(5) = 2.12$ W

$P_2 = i_2{}^2R_2 = (0.685)^2.(4) = 1.8$

$P_3 = i_3{}^2R_3 = (-0.033)^2.(8) = 0.01$

(c) Power supplied by $\mathcal{E}_2$: $P_{s2} = (\mathcal{E}_2)(i_2) = (6)(0.685) =$ 4.11 W .

Power supplied by $\mathcal{E}_1$: $P_{s1} = (\mathcal{E}_1)(i_3) = (3)(-0.033) =$ -0.1 W .

Power supplied by batteries = $P_{s2} + P_{s1} = 4$ W .

Thermal power appeared in batteries = $P_1 + P_2 + P_3 = 4$ W

$\therefore$ Energy is conserved .

Notice that $\mathcal{E}_1$ supplies negative power i.e. it takes power or it converts energy, but this electric energy is not converted to thermal energy like in resistences but to probably chemical energy in the battery.

Problem 4:

A 4 watt light bulb is designed to operate at two volts across its terminals .

A resistance R is placed in parallel with the bulb and the combination is connected as shown below to a three ohm resistor and a 12 volt battery with internal resistance 1/3 ohm .

What should the value of R be if the bulb is to operate at the designed voltage .

Solution:

Taking K.V.L. (Kirchhoff's Voltage Law) around the loop we get :

$$12 = (\frac{1}{3} + 3)\ i + R_b i_b$$

Where the subscipt b refers to the bulb .

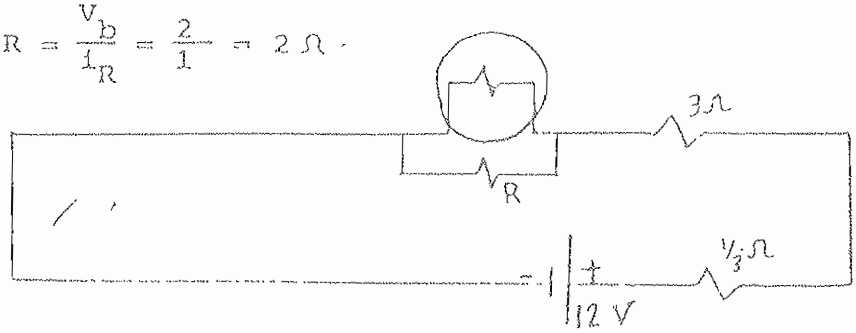
It is given that $V_b = 2V = R_b i_b$

$\therefore\ 12 = (\frac{10}{3})\ i + 2 \Longrightarrow i = 3\ A$.

$$P_b = V_b i_b \Longrightarrow i_b = \frac{P_b}{V_b} = \frac{4}{2} = 2\ A\ .$$

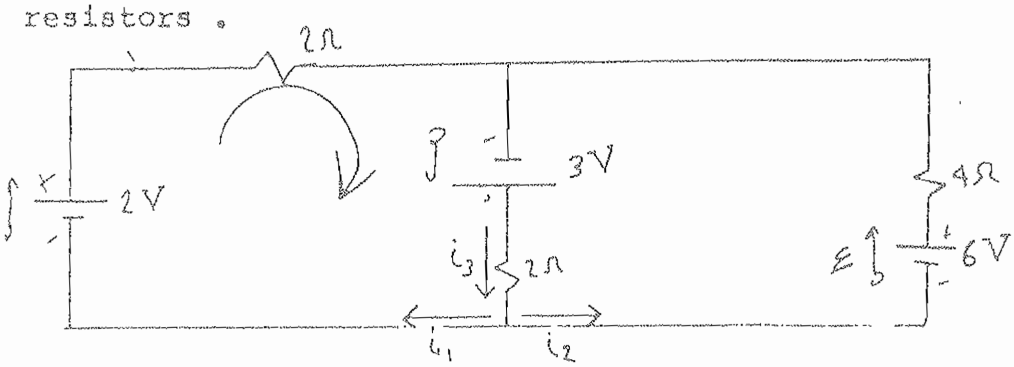
Therefore the current passing through the resistance R is $i_R = i - i_b = 3 - 2 = 1\ A$ by the K.C.L. (Kirchhoff's Current Law). But the voltage around R is 2 volts.

$\therefore\ R = \frac{V_b}{i_R} = \frac{2}{1} = 2\,\Omega$.



Problem 5:

Find in the circuit shown below the direction and magnitude of the currents through each of the three resistors .



Solution:

Writing K.V.L. to the two 'windows' we get :

$$2 + 3 = 2i_1 + 2i_3 = 5$$

$$6 + 3 = 4i_2 + 2i_3 = 9$$

Besides, using K.C.L. at the node A, we get :

$$i_1 + i_2 = i_3$$

Substituting these 3 equations we get :

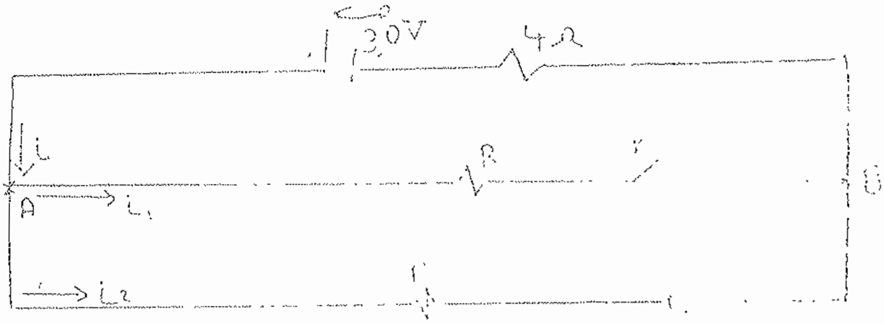
$$i_1 = 0.6 \text{ A},\ i_2 = 1.3 \text{ A} \ \&\ i_3 = 1.9 \text{ A}$$

The direction of the currents are as shown on the figure (note : The directions of the currents are not given).

Problem 6 :

A battery of emf of 30 V and internal resistance of 4 ohms is connected in a circuit as shown in the figure. When the key K is open the currentmeter reads 3A. When the key K is closed the current meter reads 1.5 A.

Calculate the resistances r and R, and calculate the difference in potential across the battery in each case. The current meter has zero resistance.



Solution:

When K is open we have the main current = i = 3 A.

When K is open, no current passes through R & R can be taken out of the circuit.

Taking K.V.L. around the circuit, we have :

$$30 = 4i + ri = i(4 + r) = 3(4 + r)$$

$\therefore\ r = 6\,\Omega$

Note : We assume that the resistance of the ammeter is zero.

When K is closed the current through $r = i_2 = 1.5$ A.

Therefore the voltage across AB is :

$$V_{AB} = i_2 r = (1.5).(6) = 9 \text{ V}$$

But $V_{AB} = 30 - 4i = 9 \Longrightarrow i = 5.25$ A.

Taking the node at A & applying K.C.L. we get :

$$i = i_1 + i_2 \Longrightarrow i_1 = i - i_2 = 5.25 - 1.5 = 3.75\text{A}.$$

$$V_{AB} = i_1 R \Longrightarrow R = \frac{V_{AB}}{i_1} = \frac{9}{3.75} = 2.4\,\Omega$$

The voltage around the battery is V_{AB} (note: the internal resistance of the battery must be included when measuring the potential voltage around the battery).

When K is open :

$$V_{AB} = 30 - 4(i) = 30 - 4(3) = 18 \text{ V}.$$

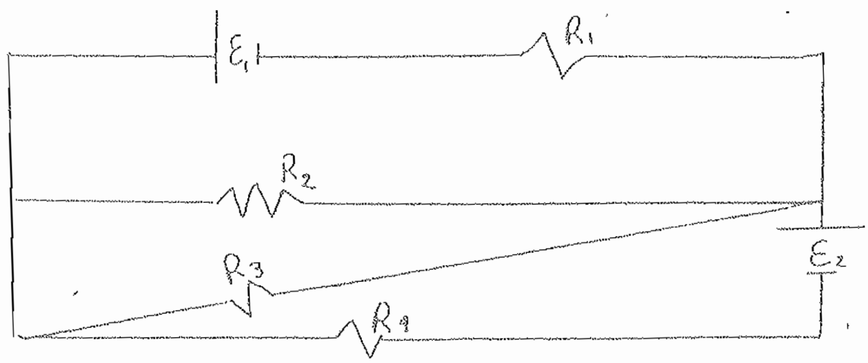
When K is closed : $V_{AB} = 9$ V as calculated .

Problem 7 :

$\mathcal{E}_1 = 10$ V.
$\mathcal{E}_2 = 2$ V.
$R_1 = 10\,\Omega$
$R_2 = 6\,\Omega$
$R_3 = 2\,\Omega$
$R_4 = 15\,\Omega$

a) Find the current in each resistor .

b) How much power is delivered by $\mathcal{E}_1$? by $\mathcal{E}_2$?

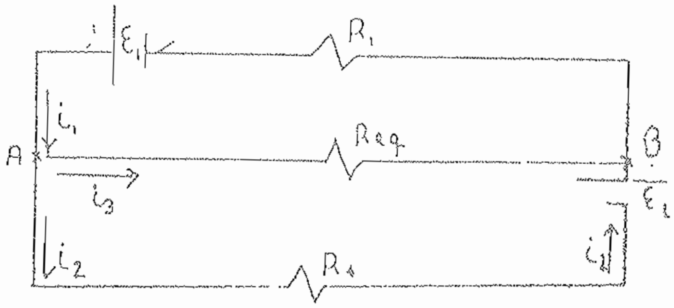


Solution:

(a) Obviously we can exchange R_2 & R_3 with their equivalent resistance because they are in parallel .

$$Req = \frac{R_2 R_3}{R_2 + R_3} = \frac{(6)(2)}{6 + 2} = 1.5\ \Omega$$

The circuit becomes :



Taking the K.V.L. around the two windows we get :

$$2 = -1.5\ i_3 + 15\ i_2$$

$$10 = 1.5\ i_3 + 10\ i_1$$

Besides, taking the K.C.L. at the node A :

$$i_1 = i_2 + i_3$$

Solving for these three equations & three unknowns we get :

$i_1 = 0.896$ A $i_2 = 0.203$ A & $i_3 = 0.693$ A .

Current through R_1 is $i_1 = 0.896$ A

" " R_4 " $i_2 = 0.203$ A

" " $R_2 = \frac{V_{AB}}{R_2} = \frac{(R_{eq})(i_3)}{R_2} =$

$\frac{(1.5).(0.693)}{6} = 0.173$ A

Current through $R_3 = \frac{V_{AB}}{R_3} = \frac{(1.5).(0.693)}{2} = 0.519$ A

(b) $P_1 = (\varepsilon_1)(i_1) = (10).(0.896) = 8.96) = 8.96$ W

$P_2 = (\varepsilon_2).(i_2) = (2).(0.203) = 0.406$ W .

Problem 8 :

The capacitor in the circuit shown has C = 5 uf and is charged to a potential difference of 100 V. R is equal to 10^6 . If a dielectric of constant k = 2 is now inserted into the capacitor and then the switch S is closed find :

a) The potential across R at t = 0.

b) The current at t = 0.1 second.

Solution:

(a) $C_o = \frac{C_K}{K} \Longrightarrow C_K = K \times C_o = 2 \times 5 = 10\ \mu F = 10^{-5}$ F

But q remains the same on the capacitor before discharging (i.e. when a dielectric is inserted) (t = 0).

$$C_o V_o = q = C_K V_K \Longrightarrow (5)(100) = (10)(V_K) \Longrightarrow$$

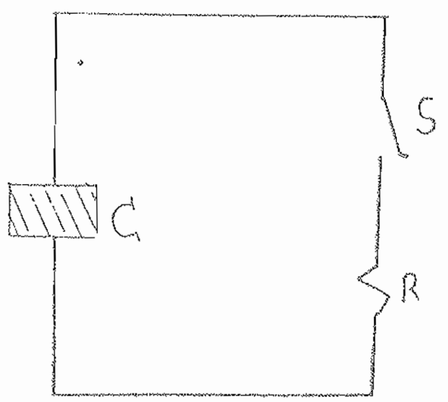
$$V_K = 50 \text{ V} .$$

(b) When discharging, the expression for i is :

$$i = \frac{V}{R} e^{-t/Rc} = \frac{50}{10^6} . e^{\left(-\frac{0.1}{10^6 \times 10^{-5}}\right)}$$

$$= (5 \times 10^{-5})\, e^{-0.01} = (5 \times 10^{-5}).(1.01)$$

$$= 5.05 \times 10^{-5} \text{ A} .$$



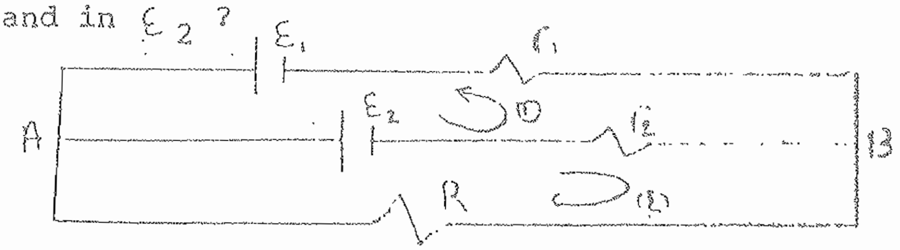
Problem 9 :

In the following circuit $\mathcal{E}_1 = 8$ volts, $\mathcal{E}_2 = 5$ volts, $r_1 = 10$ ohms and $i_1 = i_2 = 0.5$ amperes .

a) Find the resistances of the resistors r_2 and R.

b) At what rate heat generated in r_1, r_2, and R?

c) At what rate is electrical energy generated in $\mathcal{E}_1$ and in $\mathcal{E}_2$?



Solution :

(a) $V_{AB} = \mathcal{E}_1 - i_1 r_1 = 8 - (0.5).(10) = 3$ V

$V_{AB} = \mathcal{E}_2 - i_2 r_2 \Longrightarrow 3 = 5 - (0.5).(2) \Longrightarrow r_2 = 4\,\Omega$

Taking the K.C.L. at node A

$$i_1 + i_2 = i_3 \Longrightarrow i_3 = 0.5 + 0.5 = 1 \text{ A}$$

$$V_{AB} = i_3 R \Longrightarrow R = \frac{V_{AB}}{i_3} = \frac{3}{1} = 3\,\Omega$$

(b) The rate of heat generated is the power generated .

In r_1 : $P_1 = i_1^2 r_1 = (0.5)^2.(10) = 2.5$ W

In r_2 : $P_2 = i_2^2 r_2 = (0.5)^2.(4) = 1$ W

In R : $P = i_3^2 R = (1)^2 .(3) = 3$ W

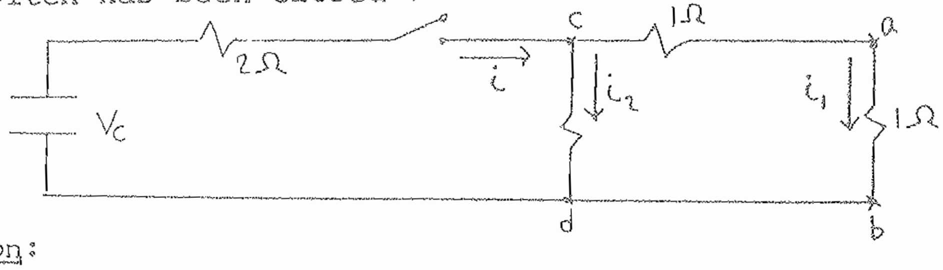
(c) At $\mathcal{E}_1$: $P = (\mathcal{E}_1)(i_1) = (8).(0.5) = 4$ W

At $\mathcal{E}_2$: $P = (\mathcal{E}_2)(i_2) = (5).(0.5) = 2.5$ W

Note that the energy (or rate of energy) generated at the generators is consumed in the resistances , showing the conservation .

Problem 10:

The capacitor in the circuit has been charged so that the voltage V_c across it is 10 V. The switch is closed at t = 0, what is V_{ab} as a function of time after the switch has been closed ?



Solution:

For the figure to be familiar to us , let us take the equivalent resistance of the resistances shown .

$$Req = 2 + \frac{(1 + 1)(1)}{(1 + 1)+(1)} = 2 + 0.66 = 2.66$$

After the switch is closed the circuit becomes equivalent to :

i decays according to the expression :

$$i = \frac{V}{Req} e^{-\frac{t}{ReqC}} = \left(\frac{10}{2.66}\right) e^{-\frac{t}{(2.66)C}} = 3.75\ e^{-\frac{0.375\ t}{C}}$$

Let the current that passes through ab be i_1 & let the current passing through cd be i_2 as shown in the figure.

$V_{cd} = i_2 = 2i_1$

Applying K.C.L. at node C :

$$i = i_1 + i_2 = 3i_1 \Longrightarrow i_1 = \frac{i}{3}$$

or $i_1 = 1.25\ e^{-\frac{0.375\ t}{C}}$

$V_{ab} = (i_1)(1) = i_1 = 1.25\ e^{-\frac{0.375\ t}{C}}$ V .

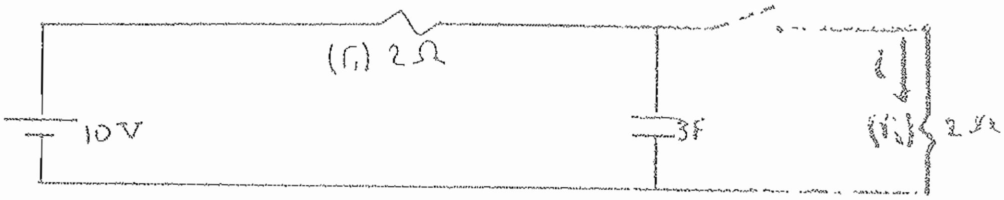
Problem 11:

The switch in the circuit is closed at $t = 0$ after a steady value of the voltage has been reached at the left side of the circuit .

a. Give the initial value of the current i

b. Give the value of the current at $t = \infty$

c. Find the initial value of $\frac{di}{dt}$ (first derivative of i).



Solution:

Initially, when the switch was upon for a long time $V_c = 10$ V. because the voltage across the r_1 was zero due to the absence of current (because of the capacitor).

(a) $i = \frac{V_{ab}}{r_2} = \frac{10}{2} = 5$ A.

Notice that V_{ab} is 10 V only at $t = 0$, then eventually V decreases .

(b) When $t = \infty$, there is no current through the capacitor. Taking K.V.L. around the whole loop (without the capacitor) we get :

$$10 = (2 + 2)\ i \Rightarrow i = \frac{10}{4} = 2.5 \text{ A.}$$

(c) At a time $t > 0$ ($t \neq \infty$) the current i is supplied both from the capacitor & the battery but at $t = 0$ (initially) all the current is supplied by the capacitor ($V_c = 10$) while the battery to supply current must pass it through the resistor thus decreasing the potential ($V_b < 10$).

So, we can forget the left hand side of the circuit for the moment.

The expression of i for the right hand side circuit is:

$$i = \frac{V}{R} e^{-t/Rc} = \frac{10}{2} e^{-t/(2)(3)} = 5 e^{-0.167\ t}.$$

$$\frac{di}{dt} = (-0.167).(5)e^{-0.167\ t} = -0.83\bar{3}e^{-0.167\ t}$$

at $t = 0$, $di/dt = -0.833$ A/s.

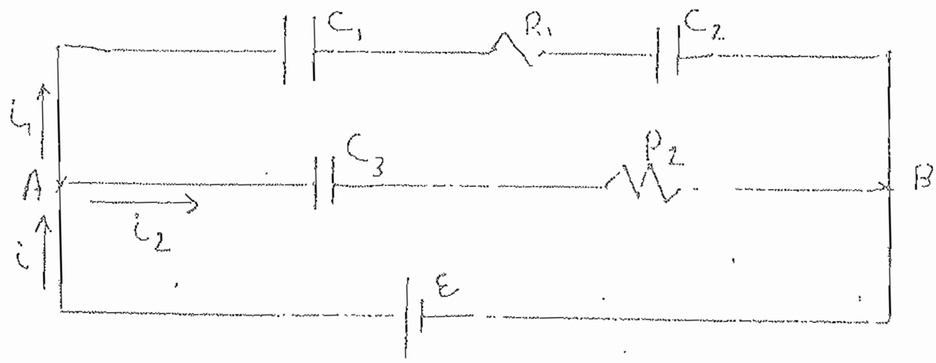
<u>Problem 12 ;</u>

In the following circuit $\mathcal{E} = 12$ volts, $R_1 = 10^6$ ohms, $R_2 = 2 \times 10^6$ ohms, $C_1 = C_2 = 4$ micro Farads, and $C_3 = 2$ microfarads.

a) Find the charges on each capacitor as a function of time.

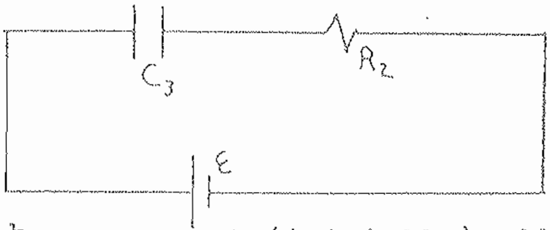
b) Deduce the value of the charges for $t = \infty$.

c) At $t = \infty$ capacitor C_3 is filled with a dielectric with a dielectric constant $K = 2$. What happens to the charges on each capacitor?



Solution:

(a) First, let us work with the lower loop.



Assume that at $t = 0$ (initially) all the capacitors are uncharged.

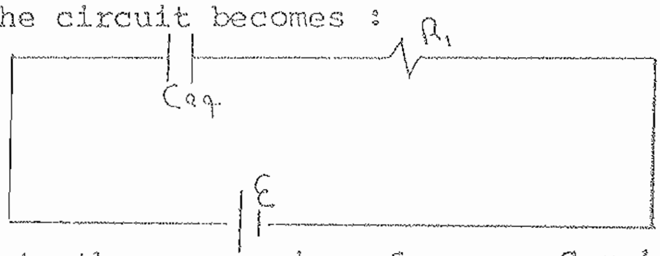
At time t, the expression of q_3 on C_3 is given as :

$$q_3 = C_3 \, \varepsilon \cdot (1 - e^{-t/R_2C_3})$$
$$= (2 \times 10^{-6}).(12).(1 - e^{-t/4})$$
$$= (2.4 \times 10^{-5}).(1 - e^{-0.25\, t})$$

Now let us take the upper line. To make the circuit familiar to us, we take the equivalent of C_1 & C_2.

$$Ceq = \frac{C_1C_2}{C_1 + C_2} = 2 \;\mu F.$$

Thus, the circuit becomes :



At time t, the expression of q_{eq} on Ceq is given as :

$$q_{eq} = Ceq\, \varepsilon \,.\, (1 - e^{-\frac{t}{R_1Ceq}})$$
$$= (2 \times 10^{-6}).(12).(1 - e^{-t/2})$$
$$= (2.4 \times 10^{-5}).(1 - e^{-0.5\, t}).$$

The charge on Ceq is the same as

the " on C_1 " " " "

" " on C_2 because C_1 & C_2 are connected in series.

$$\therefore q_1 = q_2 = q_{eq} = (2.4 \times 10^{-5}).(1 - e^{-0.5\, t})$$

(b) as $t \longrightarrow \infty$ $e^{-0.5\ t}$ & $e^{-0.25\ t}$

approach zero, so putting these values in the equations just derived we get :

$$q_3 = 2.4 \times 10^{-5}\ C$$

$$q_1 = q_2 = 2.4 \times 10^{-5}\ C.$$

(c) First of all, if a dielectric is inserted in C_3; C_1 & C_2 are not affected . C_1 & C_2 depend on $\mathcal{E}$.(review the derived equations for q_1 & q_2 where q_1 & q_2 are independent of C_3)

$\therefore$ $q_1 = q_2 = 2.4 \times 10^{-5}$ C.

For C_3: $CV = q$ but V is constant (12 V). If we add a dielectric of $K = 2$ C doubles $\Longrightarrow$ q must be doubled to keep V constant .

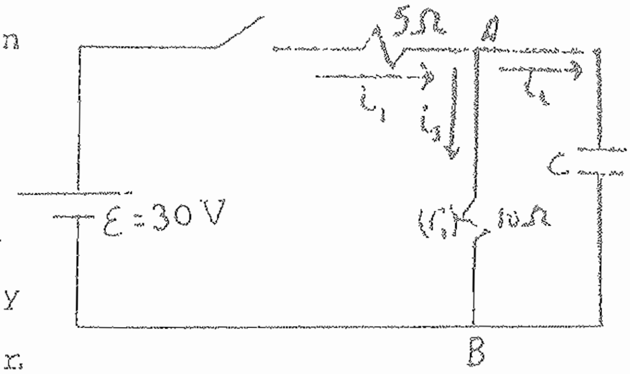
$$q_3 = 2 \times 2.4 \times 10^{-5} = 4.8 \times 10^{-5}\ C\ .$$

Problem 13:

Consider the following circuit with C = 200mf and $\mathcal{E}$ = 30 V.

a. Find the currents i_1, i_2, i_3 when the switch is closed (t = 0) and at a very long time after that (t = ∞)

b. If the switch is then opened what will the currents be at that instant (t = 0) and what values will they have one second later.



Solution:

(a) At t = 0, C acts as a short circuit therefore all the current passes through the capacitor & no current passes through the 10 Ω resistor.

∴ $i_3 = 0$.

$i_1 = \frac{\mathcal{E}}{5} = \frac{30}{5} = 6$ A.

Taking K.C.L. at node A :

$i_1 = i_2 + i_3 \Longrightarrow i_2 = i_1 - i_3 = 6 - 0 = 6$ A.

At t = ∞, C acts as an open circuit, therefore no current passes through the capacitor.

Applying the K.V.L. around the loop (not taking the capacitor).

$$30 = (5 + 10)\, i \Longrightarrow i = 2 \text{ A}.$$

Therefore $i_2 = 0$ & $i_1 = i_3 = 2$ A.

(b) The resistor of 5Ω is opened & there is no current passing through that resistance at any time after the switch is opened .

∴ $i_1 = 0$ (Both at t = 0 & at t = 1)

Before opening the switch

$V_c = V_{ab} = (i_3)(10) = 20$ V

Taking the K.C.L. at node A

$$i_1 = i_2 + i_3 \Longrightarrow 0 = i_2 + i_3 \Longrightarrow i_2 = -i_3$$

i.e. the current i_2 changes direction when the switch is closed .

When discharging the expression for i_2 is :

$$i_2 = -\frac{V_{AB}}{r_1} e^{-t/r_1 c} = -\frac{20}{10} e^{-t/2} = -2e^{-0.5\ t}$$

At t = 0 $\quad i_2 = -2$ A

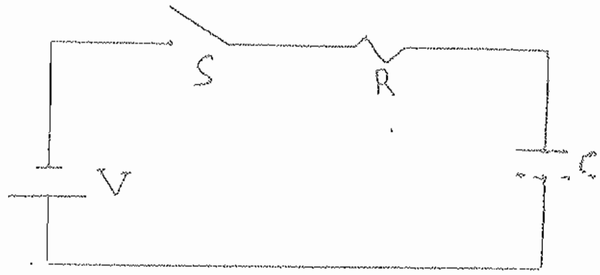
$i_3 = -i_2 = 2$ A

At t = 1s $\quad t_2 = -2\, e^{-0.5} = -1.21$ A

$i_3 = -i_2 = 1.21$ A.

Problem 14:

A capacitor C = 1 uF is suddenly connected to a battery of V = 100 volts through a resistor R = 100 Ω as shown in the figure. After how long will the capacitor be charged to a potential of 75 volts ?



Solution:

The expression of the charge q on the capacitor is given as :

$$q = CV(1 - e^{-t/Rc})$$
$$= (100\ C).(1 - e^{-t/10^{-4}})$$
$$= (100\ C).(1 - e^{-10^4 t})$$

But $V = \frac{q}{c} = 100\ (1 - e^{-10^{4t}})$

If V = 75

$\therefore\ 0.75 = 1 - e^{-10^{4t}} \Longrightarrow 0.25 = e^{-10^{4t}}$

$\ln(0.25) = -10^4 t \Longrightarrow t = 1.386 \times 10^{-4}$ s .

Problem 15 :

Consider the following circut for charging a capacitor

$R_1 = 10\,\Omega$

$R_2 = 10\,\Omega$

$C = 10^{-2}$ F

$\mathcal{E} = 10$ V

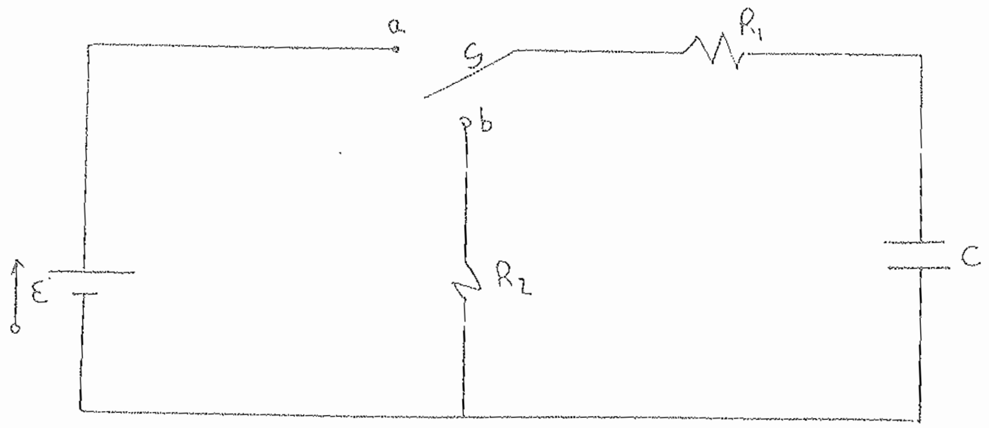
a) What are the current and charge on the capacitor 0.05 sec after switch S is closed on terminal a .

b) What will these values be after 10 sec.

c) If we wait a long time and then throw the switch onto terminal b.

What will be the charge on the capacitor and current in the circuit after 0.05 sec. after the switch is closed on b.

d) What will these values be after 10 sec .



Solution:

(a) Assume that the capacitor is initially uncharged.

When the switch is closed at a, R_2 is opened & for the moment we can assume R_2 out of the circuit. The expression for q on the capacitor is given as :

$$q = C\mathcal{E}.\,(1 - e^{-t/R_1C})$$

$$= (10^{-2}).(10).(1 - e^{-0.05/0.1})$$

$$= (0.1).(1 - 0.606) = 3.9 \times 10^{-2}\ C.$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R_1}\,(e^{-t/R_1C})$$

$$= (\frac{10}{10}).(e^{-0.05/0.1}) = 0.606\ A.$$

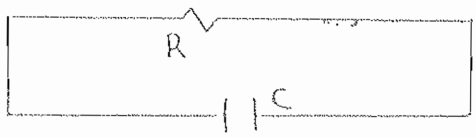
(b) $q = (0.1).(1 - e^{-100}) = 0.1\ C$

$i = (\frac{10}{10}).(e^{-100}) = 3.72 \times 10^{-44} \simeq 0\ A$

(c) Let us take the equivalent resistance of R_1 & R_2.

$R = R_1 + R_2 = 10 + 10 = 20\,\Omega$

The circuit under interest becomes:



The expression of q is given as :

$$q = q_0 e^{-t/RC}$$

$$= (0.1)\,e^{-t/0.2} = (0.1)\cdot e^{-5t}$$

At $t = 0.05$ s, $q = 7.7 \times 10^{-2}$ C

At $t = 10$ s $\quad q = (0.1).(1.92 \times 10^{-22}) = 1.92 \times 10^{-13}$C

The expression for i is given as :

$$i = \frac{dq}{dt} = -\frac{\mathcal{E}}{R}\,e^{-t/RC}$$

$$= -(\frac{10}{10}).e^{-5t} = -0.5\,e^{-5t}$$

At $t = 0.05$ s $\quad i = -(0.5).(0.778) = 0.389$ A.

At $t = 10$ s $\quad i = -(0.5).(1.92 \times 10^{-22})$

$= 9.64 \times 10^{-23}$ A.

Problem 16:

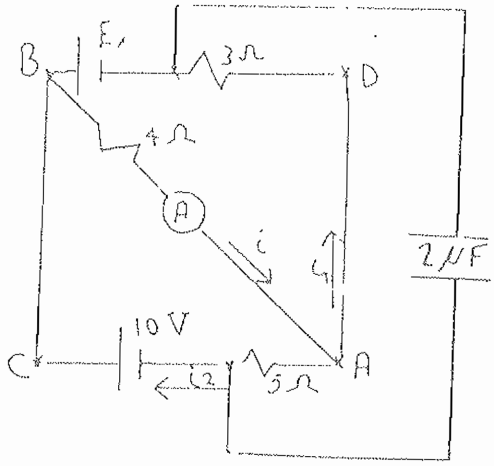
Consider the circuit shown in this figure .

If the ammeter A reads $i = 3$ amperes. Find (a long time after the circuit is connected)

a. the currents i_1 & i_2

b. the EMF F_x

c. the charge on the 2 uF capacitor .



Solution:

(a) A long time after the circuit is connected, we don't have any current in the capacitor .

Taking the K.V.L. in the loop ABC :

$$10 = 4(i) + 5(i_2) = 12 + 5i_2$$

$$\Longrightarrow i_2 = -0.4 \text{ A.}$$

The minus sign indicates that the direction of i_2 is opposite to that shown in the figure .

Taking the K.C.L. at the node A

$$i = i_1 + i_2 \Longrightarrow i_1 = i - i_2 = 3 + 0.4 = 3.4 \text{ A.}$$

(b) Using the K.V.L. in the loop ADB :

$$Ex = 4i + 3i_1 = (4).(3)+(3).(3.4) = 22.2 \text{ V.}$$

(c) $V_{FE} = (5).(0.4)+(3).(3.4) = 12.2$ V

(or $V_{FE} = Ex - 10 = 12.2$ V)

On the capacitor

$$CV = q \Longrightarrow q = (2 \times 10^{-6}).(12.2) =$$

$$2.44 \times 10^{-5} \text{ C .}$$

Problem 17:

A parallel plate capacitor has area A and separation d is connected to a battery of EMF $\mathcal{E}$ through a resistor R .

a. If the capacitance $C = 3 \times 10^{-3}$ F and $R = 1000\ \Omega$ and $\mathcal{E} = 50$ V. After how long will the charge on the capacitor reach 3/4 of its final value .

If after a long time passes a slab of dielectric of thickness d is inserted in between the plates of the capacitor so that it fills one third of the space between the plates Calculate:

b. the charge on the plates after another long period of time passes .

c. The electric field between the plates in terms of d.

Solution:

(a) The charge on the capacitor is given by the expression:

$$q = C\,\mathcal{E}.(1 - e^{-t/Rc})$$

The 3/4 of the final value is $3/4\ C\mathcal{E} = 3/4\ q_o$

$\therefore\ 3/4\ C\mathcal{E} = C\mathcal{E}.(1 - e^{-t/3}) \Longrightarrow$

$0.75 = 1 - e^{-t/3} \Longrightarrow 0.25 = e^{-t/3} \Longrightarrow$

$\ln(0.25) = \frac{-t}{3} \Longrightarrow t = -3\ \ln(0.25)$

$\therefore\ t = 4.15$ s.

(b) Assume that the capacitor is made up of two capacitors one with the dielectric (C_1) the other without the dielectric (C_2) joined in parallel . The equivalent capacitor is given as : $C^1 = C_1 + C_2 = \frac{\varepsilon_o}{d}(\frac{KA}{3} + \frac{2A}{3}) = \frac{\varepsilon_o A}{d}(\frac{K+2}{3})$.

note : If the dielectric fills one third of the space between the space then it occupies one third of the area because the thickness of the capacitor is constant).

$C = \frac{\varepsilon_0 A}{d} \implies C^1 = C(\frac{K + 2}{3})$

As we wrote :

$q = C^1 \mathcal{E} . (1 - \varepsilon^{-t/RC'})$ if t increases to infinity.

$\varepsilon^{-t/RC'}$ drops to zero .

$\therefore q = C^1 \mathcal{E} = \frac{C(2 + K)}{3}$.

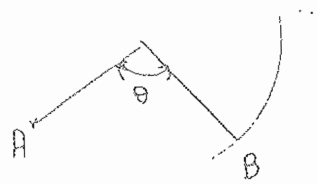
$= \frac{(3 \times 10^{-3})(2 + K)}{3} . 50 = 0.05(2 + K)$ C.

(c) Both in the dielectric & in the empty parts of the capacitor the voltage across the capacitor is the same (= 50 V) and d is constant everywhere inside the capacitor .

$\because E = \frac{50}{d}$

Problem 18 :

A metallic wire of length L = 20 cm and resistance 60 ohms is bent to form a circular loop. Find the equivalent resistance between two points on the loop making an angle $\theta = \frac{2\pi}{3}$ with the center ?



Solution:

The equival nt resistance is the two resistances (the two loops of AB i.e. below & above AB).

The length of AB (lower loop) = L/3 = 0.2/3 = 0.066 m.

The length of AB (upper loop) = 2L/3 = (2/3).(0.2) = 0.133 m.

Let the subscript 1 refer to the lower loop & the subscript 2 refer to the upper loop.

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

The resistance is proportional to the length (l)

$$\frac{r}{60} = \frac{l}{0.2} \Longrightarrow r = 300\, l$$

$$\therefore R_1 = 300\, l_1 = (300).(0.0666) = 20\,\Omega$$

$$\& \quad R_2 = 300\, l_2 = (300).(0.1333) = 40\,\Omega$$

$$\therefore R = \frac{(20)(40)}{(20 + 40)} = 13.333\,\Omega \ .$$

THE MAGNETIC FIELD

Problem 1:

a. Suppose a current is due to the motion of both positive and negative charges under the influence of an electric field. If n^+ and n^- are respectively the number per unit volume of these charges. Assuming they have the same drift velocity v_d what is the current in a wire of diameter d ?

b. If this wire is subject to magnetic field $\vec{B}$ perpendicular to the wire what is the potential developed across the wire due to the Hall effect ?

Solution:

(a) Consider a length l of the wire . The cross section of the wire is $A = \frac{\pi d^2}{4}$. Let the time to pass the charge be t (t is the same for both charges because v_d & l are the same). We have

$l = v_d t$, but $q = Vne$ where $V = lA$ (is the volume of the wire of length l. Besides, $i = \frac{q}{t}$ (let the subscript 1 refer to positive charges & subscript 2 refer to negative charges. The two currents have opposite charges but also opposite directions, so they add up.

$$i_1 = \frac{q_1}{t} = \frac{(lA).(n^+e)}{t} = \frac{(v_d t)}{t} \cdot \left(\frac{\pi d^2}{4}\right) \cdot (n^+ . e).$$

$$\Longrightarrow \quad i_1 = \frac{v_d \pi d^2 n^+ e}{4}$$

Similarly : $i_2 = \frac{v_d \pi d^2 n^- e}{4}$

$$i_T = i_1 + i_2 = \frac{\pi d^2 e\, v_d}{4} (n^+ + n^-)$$

If $/n^+/ = /n^-/ = n$ then

$$i_T = \frac{\pi d^2 e\, v_d\, n}{2} .$$

(b) This wire being subjected to a magnetic field, creates a force of magnetic origin that causes both the positive & the negative charges to accumulate on the same side (because as mentioned they have opposite signs in the charge & direction of motion, thus the two opposite signs cancel each other giving the same sense (direction) of force).

If the magnitudes of charges are equal then they neutralize each other & thus don't create any electric field or potential difference accoss the wire .

If the magnitudes of the charges are not equal (assume $n^+ > n^-$) then $n_{net} = (n^+ - n^-)$

But $E_H = \frac{jB}{ne} = \frac{jB}{(n^+ - n^-)e}$

$$V = E_H . d = \frac{jBd}{(n^+ - n^-)e} \quad , \quad j = \frac{4i}{\pi d^2}$$

$$\therefore V = \frac{4iB}{\pi d(n^+ - n^-)e} = \frac{d\, v_d (n^+ + n^-) B}{4(n^+ - n^-)}$$

Note: n^+ & n^- are both magnitudes i.e. they are positive.

Problem 2:

A 25 KeV electron moving horizontally enters a region of space in which a downward directed electric field $\vec{E}$ of magnitude 500 V/cm is present .

a. What are the magnitude and direction of the field of magnetic induction that will allow the electron to continue in its horizontal path ?

$$1eV = 1.6 \times 10^{-19}\ J$$

$$m_e = 9.1 \times 10^{-31}\ kg$$

$$q = 1.6 \times 10^{-19}\ C$$

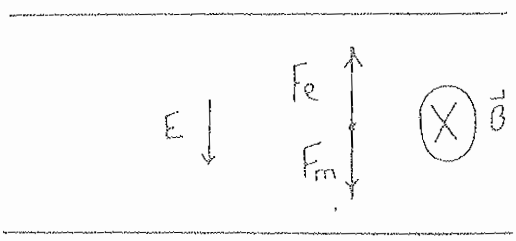
b. If the electric field is removed, describe the trajectory of the electron in the remaining magnetic field: (draw a sketch and calculate what ever is necessary :).

Solution:

(a) $K.E. = \frac{1}{2} mV^2 \Longrightarrow V = \left(\frac{2\ K.E.}{m} \right)^{\frac{1}{2}}$

$$= \left(\frac{2 \times 25 \times 10^3 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} \right)^{\frac{1}{2}} = 9.37 \times 10^7\ m/s$$

The coulombic force is upwards because it has the opposite direction of the electric field due to negative charge. If the net force will be



zero, we have to create a force Fm of opposite direction (downwards) but of equal magnitude as the coulombic force Fe .

$$Fe = Fm \Longrightarrow q\ E = qVB \Longrightarrow B = \frac{E}{V} = \frac{5 \times 10^4}{9.37 \times 10^7}$$

$= 5.33 \times 10^{-4}$ Tesla = 5.33 gauss .

By the right hand rule the direction of the magnetic field is into the paper as shown in the figure .

(b) The only force acting on the electron is the magnetic force whose direction is always perpendicular to the direction of the velocity, thus no work is done and the trajectory is a circle where Fm is always directed to the center .

$$F_m = ma = \frac{mV^2}{r} \qquad \text{but } F_m = eVB$$

$$\therefore eB = \frac{mV}{r} \Longrightarrow r = \frac{mV}{eB} = \frac{(9.11 \times 10^{-31}) \cdot (9.37 \times 10^{7})}{(1.6 \times 10^{-19}) \cdot (5.33 \times 10^{-4})}$$

$= 1$ m .

Problem 3 :

A length l of wire carries a current i. Show that if the wire is formed into a circular coil, the maximum torque in a given magnetic field is developed when the coil has 1 turn only & the max. torque has the value

$$\tau = \frac{1}{4\pi} \cdot l^2 iB.$$

Solution:

Let the number of turns be n .

The periphery of the circle is $\frac{1}{N}$

But the periphery is also equal to $2\pi \cdot r$

$\therefore r = \frac{1}{2\pi N}$ $\therefore A = \pi r^2 = \frac{1^2}{4\pi N^2}$

The torque is given by :

$T = NiAB \sin\theta$

For maximum torque the magnetic field is perpendicular to the plane of the area i.e. $\sin\theta = 1$

$\therefore T = N \left(\frac{1^2}{4\pi \cdot N^2} \right) iB = \frac{1^2 iB}{4\pi \cdot N}$

The torque is maximum when N = 1 i.e. when the coil has one turn .

$\therefore T = \frac{1^2 iB}{4\pi}$

Problem 4 :

Figure shows an arrangement used by Dempster to measure the masses of ions. An ion of mass M and charge +q is produced essentially at rest in source S, a chamber in which a gas discharge is taking place The ion is accelerated by potential difference V & allowed to enter a magnetic field $\vec{B}$. In the field it moves in a semicircle, striking a photographic plate at distance x from the entry slit is recorded. Show that the mass M is given by

$$M = \frac{B^2 q x^2}{8V}$$

Solution:

When the particle just enters the magnetic field its kinetic energy is equal to the energy gained by the potential difference .

$$\therefore\ qV = \tfrac{1}{2} Mv^2 \Longrightarrow v^2 = \frac{2qV}{M}$$

In the magnetic field the magnetic force is equal to the mass times acceleration.

$$F_m = Ma \Longrightarrow qvB = \frac{Mv^2}{r}$$

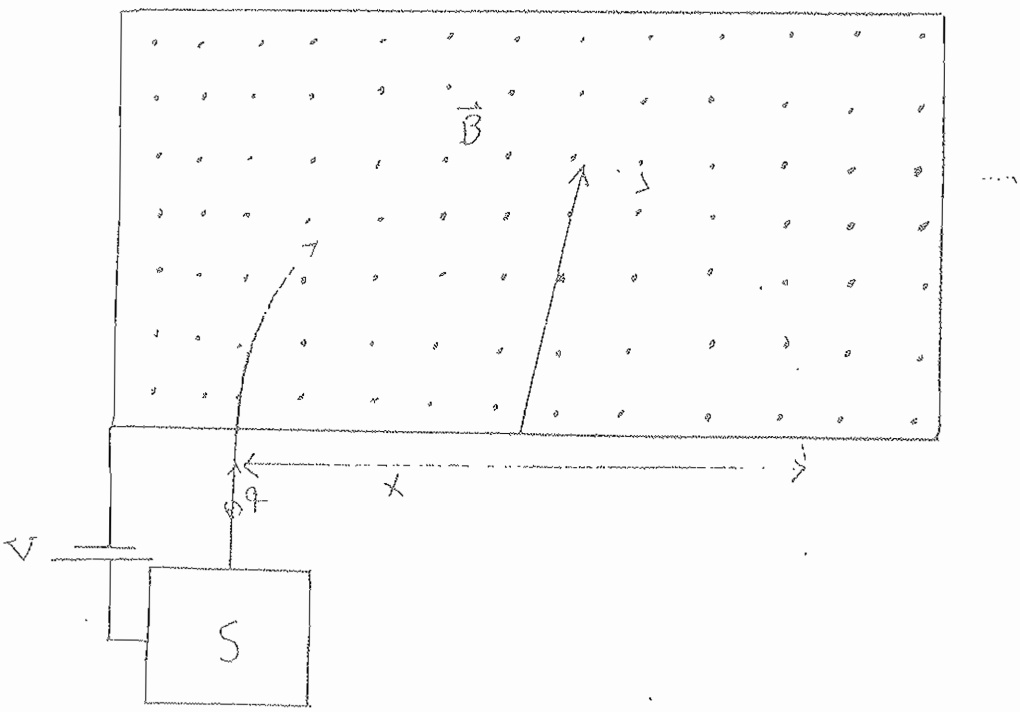
$$\Longrightarrow v^2 = \frac{r^2q^2B^2}{M^2}$$

Substituting these two equations :

$$\frac{2qV}{M} = \frac{r^2q^2B^2}{M^2} \Longrightarrow M = \frac{r^2qB^2}{2V}$$

But $r = x/2 \Longrightarrow r^2 = \frac{x^2}{4}$

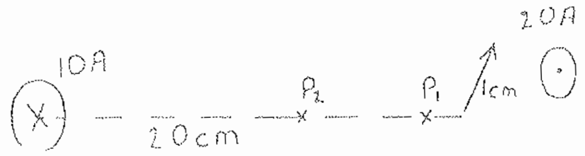
$$\therefore\ M = \frac{x^2qB^2}{8V}$$



AMPERE'S LAW

<u>problem 1</u>:

An infinite long wire carries a current of 10A and is parallel to a thin conducting cylindrical shell of radius 1 cm and carrying a current of 20A in the opposite direction, The wire is 20 cm away from the axis of the shell .



a. Find the magnetic field at the point P a distance of 0.5 cm from the axis of the shell along the line joining the axis to the wire and perpendicular to both as shown .

b. Find the magnetic field at the point P_2 a distance of 2 cm away from the axis of the shell along the same line .

c. What are the answers to (a) and (b) above if the current in the shell is reversed in direction ?

<u>Solution</u>:

(a) We first find the magnetic field at P_1 due to the wire then due to the shell , then vectorially add them .

For the wire: Using Ampere's law :

$$\oint \vec{B_1}\vec{dl} = \mu_0 i_1 \Longrightarrow B_1(2\pi r_1) = \mu_0 i_1$$

$$\Longrightarrow B_1 = \frac{\mu_0 i_1}{2\pi r_1} = \frac{(2 \times 10^{-7}).(10)}{(20 - 0.5).(10^{-2})} = 1.025 \times 10^{-5} \text{Tesla}$$

For the shell : Using Ampere's law :

$$\oint \vec{B}\,\vec{dl} = \mu i \Longrightarrow B(2\pi r) = 0 \longrightarrow B = 0$$

i_2 is zero because we took the concentric circle of $r = 0.5$ cm from the center of the shell, through this area no current passes.

$\therefore \vec{B} = \vec{B}_1 + \vec{B}_2 = 1.025 \times 10^{-5} + 0 = 1.025 \times 10^{-5}$ Tesla

By the right hand rule the direction [illegible] downwards.

(b) For the wire: As we arrived in part (a)

$$B_1 = \frac{\mu_0 i_1}{2\pi r_1} = \frac{(2 \times 10^{-7}).(10)}{(20 - 2).(10^{-2})} = 1.11 \times 10^{-5} \text{ Tesla}$$

For the shell: Taking the concentric circle of $r = 2$ cm as the closed line integral:

$$\oint \vec{B}_2 d\vec{l} = \mu_0 i_2 \Rightarrow B_2(2\pi r_2) = \mu_0 i_2$$

$$\Rightarrow B_2 = \frac{\mu_0 i_2}{2\pi r_2} = \frac{(2 \times 10^{-7}).(20)}{0.02} = 2 \times 10^{-4} \text{ Tesla}$$

By the right hand rule both $\vec{B}_1$ & $\vec{B}_2$ are directed downwards, therefore $\vec{B}$ is directed downwards.

$\vec{B} = \vec{B}_1 + \vec{B}_2 = 1.11 \times 10^{-5} + 2 \times 10^{-4} = 2.11 \times 10^{-4}$ Tesla

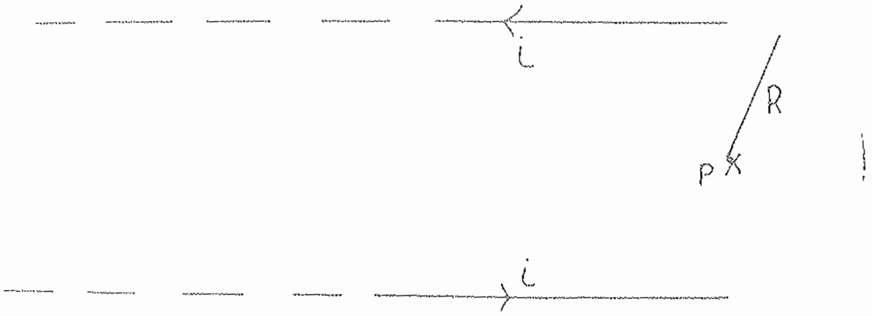
(c) The answers in part (a) do not change because i_2 does not enter in the calculations. As for part (b), $\vec{B}_2$ will be directed upwards & taking upwards as the positive direction we have:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = -1.11 \times 10^{-5} + 2 \times 10^{-4} = 1.889 \times 10^{-4}$$

Tesla.

Problem 2:

A long wire is bent into the shape shown in the figure. What is the magnitude and direction of the magnetic field at the point P ?



Solution:

Let us find the magnitude of the magnetic field, at P due to the two straight wires, $\vec{B}_1$ & $\vec{B}_2$; then due to the semi-circle $\vec{B}_3$. Note that all the three vectors point vertically upwards & that $\vec{B}_1 = \vec{B}_2$ due to symmetry .

Using the law of Biot & Savart for $\vec{B}_1$:

$$dB_1 = \frac{\mu_0 i \, dx \sin\theta}{4\pi r^2}$$

$$B_1 = \int dB_1 = \frac{\mu_0 \cdot i}{4\pi} \cdot \int \frac{\sin\theta \, dx}{r^2}$$

But $r = (x^2 + R^2)^{\frac{1}{2}}$

& $\sin\theta = \sin(\pi - \theta) = \dfrac{R}{(x^2+R^2)^{\frac{1}{2}}}$

$$\therefore B_1 = \frac{\mu_0 \cdot i}{4\pi} \cdot \int_0^{\infty} \frac{R\,dx}{(x^2+R^2)^{\frac{3}{2}}} = \frac{\mu_0 i}{4\pi R} \cdot \left[\frac{x}{(x^2+R^2)^{\frac{1}{2}}} \right]_0^{\infty}$$

$$= \frac{\mu_0 i}{4\pi R}$$

But $B_2 = B_1 \Longrightarrow B_2 = \dfrac{\mu_0 i}{4\pi R}$

For the semi circle

$$dB_3 = \frac{\mu_0 i \, d l \sin\theta}{4 \pi R^2} \quad , \text{ where } \theta = 90^\circ$$

$$\therefore B_3 = \int dB_3 = \frac{\mu_0 i}{4 \pi R^2} (\pi R) = \frac{\mu_0 i}{4R}$$

$$\therefore B = B_1 + B_2 + B_3 = \frac{\mu_0 i}{4 \pi R} + \frac{\mu_0 i}{4 \pi R} + \frac{\mu_0 i}{4R}$$

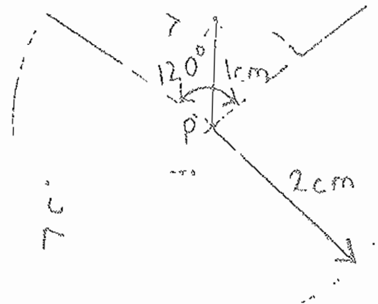
$$= \frac{\mu_0 i}{4R} \left(\frac{1}{\pi} + \frac{1}{\pi} + 1 \right) = \frac{\mu_0 i (2 + \pi)}{4 \pi R}$$

$$= \frac{5.14 \times 10^{-7} \, i}{R} \quad \text{Tesla}$$

Problem 3:

What is the magnetic field at the point P in the figure shown .

Give magnitude and direction.



Solution:

The magnetic field at P will be determined first due to the upper arc (1) & then due to the lower arc (2).

Using the law of Biot & Savart for the upper arc .

$$dB_1 = \frac{\mu_0 i \, dl \sin\theta}{4\pi r_1^2} \quad , \text{ where } \theta = 90^\circ$$

$$\therefore B_1 = \int dB_1 = \frac{\mu_0 i}{4\pi \cdot r_1^2} \cdot \left(\frac{2\pi \cdot r_1}{3}\right) = \frac{\mu_0 i}{6 r_1}$$

$$= \frac{(4\pi \times 10^{-7}).(i)}{6 \times 10^{-2}} = 2.09 \times 10^{-5} \, i \text{ T.}$$

By the right - hand - rule $\vec{B}_1$ is directed into the page.

Working similarly for $\vec{B}_2$:

$$B_2 = \frac{\mu_0 i}{4\pi \cdot r_2^2} \left(\cdot \frac{4\pi \cdot r_2}{3}\right) = \frac{\mu_0 i}{3 r_2} =$$

$$\frac{(4\pi \times 10^{-7}).(i)}{3 \times 2 \times 10^{-2}} = 2.09 \times 10^{-5} \, i \text{ Tesla} .$$

By the right hand rule B_2 is directed into the page .

$\vec{B} = \vec{B}_1 + \vec{B}_2 = 4.18 \times 10^{-5}$ i Tesla , directed into the page .

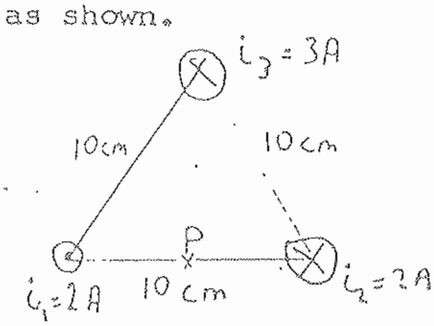
Problem 4 :

Three parallel wires carry currents i_1, i_2 and i_3 and are all perpendicular to the paper as shown.

i_1 is equal to 2 A and is directed out of the paper .

i_2 is equal to 2 A and is directed into the paper .

i_3 is equal to 3 A and is directed into the paper.



a) Find $\overline{B}$ at the point P midway between the wires with currents i_1 and i_2. Sketch it on the figure.

b) If we take a circle centered at i_3 and of radius 8 cm what is $\oint \vec{B}.\vec{dl}$ around this circle if we integrate in a counter clockwise direction .

Solution:

(a) For a long wire, we can find the magnetic field at a distance r from it by the Ampère's law taking a circle centered at the wire and having a radius r

$$\int \vec{B}\,\vec{d\,l} = B\int dl = B(2\pi \cdot r) = \mu_0 i \Longrightarrow B = \frac{\mu_0 i}{2\pi r}$$

Superimposing the three magnetic fields $\vec{B_1}$, $\vec{B_2}$ & $\vec{B_3}$ due to i_1, i_2 & i_3 respectively at P we get B_T .

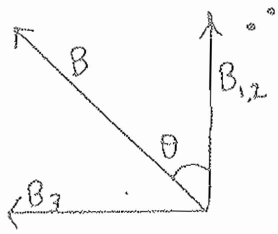
$$B_1 = \frac{\mu_0 i}{2\pi r_1} = \frac{(2 \times 10^{-7}).(2)}{5 \times 10^{-2}} = 8 \times 10^{-6} \text{ Tesla}$$

directed upwards .

$$B_2 = \frac{\mu_0 i_2}{2\pi \cdot r_2} = 8 \times 10^{-6} \text{ Tesla directed upwards .}$$

$$B_3 = \frac{\mu_0 i_3}{2\pi \cdot r_3} = \frac{(2 \times 10^{-7}).(3)}{(8.66 \times 10^{-2})} = 6.92 \times 10^{-6} \text{ Tesla}$$

directed to the left .



$$\therefore B = (B_{1,2}^2 + B_3^{\,2})^{1/2} = 1.74 \times 10^{-5} \text{ T}$$

$$\theta = \tan^{-1}\frac{B_3}{B_{1,2}} \Longrightarrow \theta = 23.4^0$$

(b) $\oint \vec{B}.\vec{dl} = \mu_0 i_3 = (4\pi \times 10^{-7}).(3) = 3.77 \times 10^{-7}$ Wb.

Note : i_1 & i_2 lie outside the circle of radius 8 cm so only i_3 is included .

Problem 5 :

A beam of 10^{15} electrons are moving clockwise in a circle of radius 50 cm in a magnetic field of 2 Tesla directed into the paper . If the mass of the electron is 9.1×10^{-31} Kg find :

a) The velocity of the electrons .

b) The current due to the electrons .

c) The net magnetic field at the center of the circle.

Solution:

(a) $Fm = ma \Longrightarrow$

$$qVB \sin\theta = \frac{mV^2}{R}$$

$$\sin\theta = 90^\circ$$

$$\therefore V = \frac{RBq}{m} = \frac{(0.5).(2).(1.6 \times 10^{-19})}{(9.11 \times 10^{-31})} = 1.75 \times 10^{11} \text{ m/s}$$

(b) $i = q.f = q.\left(\frac{w}{2\pi}\right) = q.\left(\frac{V}{2\pi R}\right)$

$$\therefore i = \frac{(10^{15} \times 1.6 \times 10^{-19}).(1.75 \times 10^{11})}{(2\pi \times 0.5)} = 8.91 \times 10^{6} A$$

(c) Using the law of Biot & Savart for the circulating electrons (Be)

$$dBe = \frac{\mu_0\, i\, dl \sin\theta}{4\pi R^2} \quad \text{where} \quad \sin\theta = 0$$

(Take the line integral the circle along which the electrons move).

$$Be = \int dBe = \frac{\mu_0 i}{4\pi \cdot R^2}(2\pi \cdot R) = \frac{\mu_0 \cdot i}{2R}$$

$$= \frac{(4\pi \times 10^{-7}).(8.91 \times 10^{6})}{2 \times 0.5} = 11.2 \text{ T directed out}$$

But $\vec{B}$ is directed into the page .

$\therefore B_{NET} = B + Be = -2 + 11.2 = 9.2$ T directed out of the page .

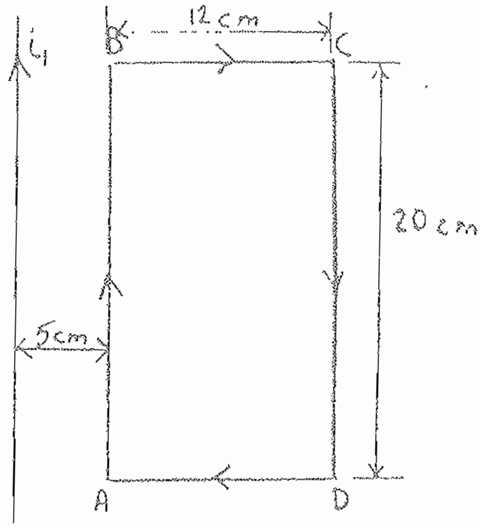
Problem 6:

For the figure below $i_1 = 3.0$ amperes and $i_2 = 4.0$ amperes .

a. Find the resultant force and the torque acting on the loop ABCD .

b. What will be the total current in the loop if current i_1 increases at a rate of 20 A .s -1 . The resistance of the loop ABCD is 12 ohms ?

c. What will be the total current in the loop if i_1 remains constant , and the loop moves with a velocity $v = 5$ m.s^{-1} in a direction parallel to i_1 ?



Solution:

(a) AB & CB have currents of opposite directions, therefore they will repel each other by equal forces , so that the resultant force on the loop is zero (due to these two forces only).

Similarly BC & AD have currents of opposite directions, so they will repel each other by equal forces, so their resultant force is zero .

Morover AD & BC (sep rately) create equal & opposite forces on BA & CD & similarly AB & CD create equal &

opposite forces on BC & AD, but all these forces cancel each other so that the loop ABCD when isolated is in equilibrium .

The wire (carrying current i_1) creates an upward force at BC but an equal but downward force on AD so that these two forces cancel each other .

The wire (i_1) & AB have the same direction of current, therefore they attract each other by the force given as

$$F_{AB} = \frac{\mu_0 \cdot l \cdot i_1 \cdot i_2}{2\pi \cdot d} = \frac{(4\pi \times 10^{-7}).(0.2).(3).(4)}{2\pi \times 0.05}$$

9.6×10^{-6} N .(to the left)

The wire (i_1 & CD have currents of opposite directions, therefore they repel each other by a force .

$$F_{CD} = \frac{\mu_0 \, l \cdot i_1 \cdot i_2}{2\pi \cdot d} = \frac{(4\pi \times 10^{-7}).(0.2).(3).(4)}{2\pi \times 0.17}$$

$= 2.82 \times 10^{-6}$ N (to the right)

Adding the two forces & taking the left direction as the positive direction

$$F = F_{AB} + F_{CD} = 9.6 \times 10^{-6} - 2.82 \times 10^{-6}$$
$$= 6.78 \times 10^{-6} \text{ N.} \quad \text{(to the left).}$$

(b) This & the following parts to be solved after studing Faraday's law .

The magnetic field due to i_1 at a distance r from the wire is given to be

$$B = \frac{\mu_0 i_1}{2\pi \cdot r}$$

The flux through the loop is given

$$\emptyset = \int \vec{B}\, \vec{dA} = \int B \cdot d\, A = \int_{0.05}^{0.17} B \,.(0.2 \times dr)$$

$$= \frac{\mu_0 i_1}{2\pi} .(0.2) \ln(\frac{17}{5}) = 4.9 \times 10^{-8}\ i_1$$

The voltage induced around the loop is :

$$V = \frac{-d\,\emptyset}{dt} = -4.9 \times 10^{-8}\ \frac{di}{dt}$$

$$= -(4.9 \times 10^{-8}).(20) = 1 \times 10^{-6}\ V$$

The induced current is

$$i = \frac{V}{R} = \frac{10^{-6}}{12} = 8.16 \times 10^{-8}\ A$$

By Lenz's law, the loop creates a magnetic field directed out of the page, therefore i is unticlockwise

$i_T = i_2 - i = 4 - 8.16 \times 10^{-8} = 3.9999999$ A

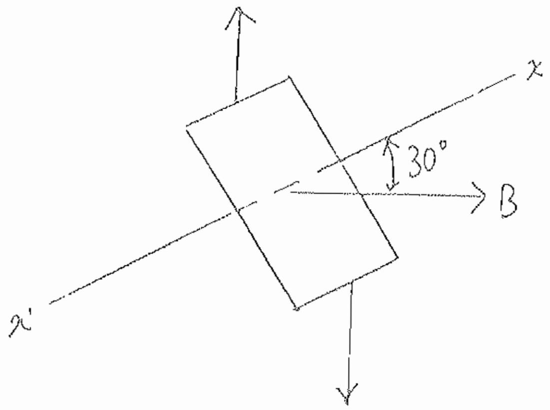
(clockwise)

(c) By moving parallel to i_1 there will be no change of flux because the flux is related to r & r remains the same here, therefore there is no induced voltage & hence no induced current .

$i_T = i_2 = 4$ A (clockwise).

Problem 7:

A solenoid of radius 1 cm and having 100 turns over a length 10 cm has a current of 1 amp. If this solenoid is placed in a uniform magnetic field of intensity 5×10^{-3} Tesla with its axis making an angle of 30° with $\vec{B}$. What is the torque $\vec{T}$ acting on the solenoid? (magnitude and direction [make a drawing !]).



Solution:

The magnetic dipole moment of the loop is given by

$$\mu = N\ i\ A = (100).(1).(\pi \times 10^{-4}) = 0.0314\ Am^2$$

$$T = \mu .\ B.\ \sin\theta = (0.0324).(5 \times 10^{-3}).(\tfrac{1}{2})$$

$$= 7.85 \times 10^{-5}\ Nm$$

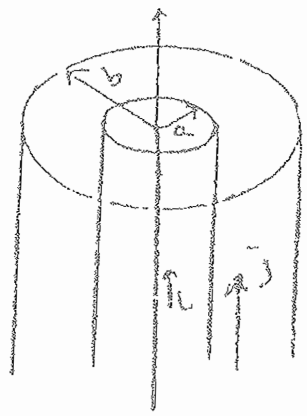
Directed into the page

Problem 8 :

A long cylindrical shell of radii a and b (a b) carries a current with a variable current density j = A/r where A is a constant. A long wire carrying a current i is placed along the axis of this shell.

a - Calculate the current through a circle of radius r (a r b)

b - Use Amperes law to find the magnetic induction B at a point inside the shell , a distance r from the axis (a r b)



Solution:

(a) The current through the shell (i_2) as a function of r is :

$$i_2 = \int j \, dA = \int_a^r \left(\frac{A}{r} \right) . (2\pi \cdot r dr)$$

$$= 2\pi A (r - a)$$

$$i_T = i_2 + i$$

$$\therefore \; i_T = 2\pi \; A(r - a) + i$$

(b) Taking as the closed line integral a concentric circle of radius r

$$\oint \vec{B} . \vec{dl} = \mu_0 i_T \implies B(2\pi \; r) = \mu_0 (i + 2\pi \; A(r-a))$$

$$\therefore \; B = \mu_0 \frac{[i + 2\pi \; A(r - a)]}{2\pi \; r}$$

Problem 9:

The electron in a hydrogen atom is in a "circular orbit" of radius r and velocity V around the proton and moving counter clockwise

a. What is the magnetic field at the position of the proton due to the electron.

b. If the proton has magnetic moment $\vec{u}$ that can point either parallel to or anti-parallel to the magnetic field.
What is the energy associated with this magnetic moment in either case ?

Solution:

(a) $f = \frac{w}{2\pi} = \frac{V}{2\pi \cdot r}$

$$i = e.f = (1.6 \times 10^{-19}).\left(\frac{V}{2\pi \cdot r}\right) = 2.55 \times 10^{-20}\ \frac{V}{r}\ A .$$

$$dB = \frac{\mu_o i\ d\ l \sin\theta}{4\pi\ r^2}$$ where $\theta = 90^o$ & $\vec{B}$ passes through the axis of the circle .

$$B = \int dB = \frac{\mu_o i}{4\pi r^2} \cdot \int dl = \frac{i(2\pi r)}{4\pi \cdot r^2}$$

$$= \frac{\mu_o i}{2r} = \frac{(2\pi \times 10^{-7}).(2.55 \times 10^{-20}.\ V)}{2r^2} = \frac{8\times10^{-27}\ V}{r^2}$$ Tesla .

(b) When μ is parallel to $\vec{B}$

$$W_o = -\vec{\mu}.\vec{B} = -\mu.B.\cos o = -\mu.B = -8\times10^{-27}\mu\ \frac{V}{r^2}$$

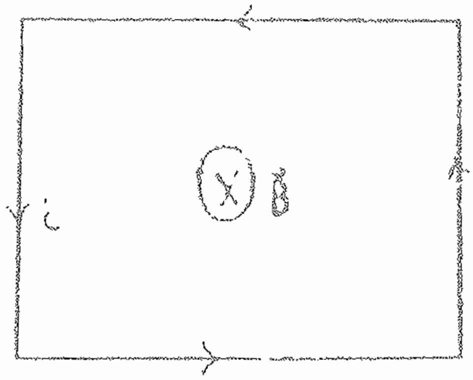
When μ is antiparallel to $\vec{B}$

$$W = -\vec{\mu}.\vec{B} = -\mu.B.\cos 180^o = \mu.B = 8\times10^{-27}\mu\frac{V}{r^2}$$

FARADAY'S LAW OF INDUCTION

Problem 1:

A loop of metallic wire of area $0.20\ m^2$ and resistance 4.0×10^{-2} ohms lied in the plane of the paper. Perpendicularly to the paper and into it, there is a magnetic field which changes with time. The magnitude of the field is $6.0 - 3.0 \times 10^{-3}\ t^2$, in Tesla where t is in seconds .



a. What is the induced emf in the wire at t = 2.0 seconds.

b. What is the current in the wire at t = 2.0 s ? What is its direction ? Explain .

c. Calculate the work done by the induced emf between t = 0, and t = 1.0 s.

Solution:

(a) $\phi = \oint \vec{B} d\vec{A} = BA = (6 - 3 \times 10^{-3}\ t^2).(0.2)$

$= (1.2 - 6 \times 10^{-4}\ t^2)$.

The induced voltage is :

$V = \frac{-d\phi}{dt} = + 2 \times 6 \times 10^{-4}\ t = 1.2 \times 10^{-3}\ t\ \ V$

At t = 2 s; $V = (1.2 \times 10^{-3}).(2) = 2.4 \times 10^{-3}\ V$

(b) $i = \frac{V}{r} = \frac{2.4 \times 10^{-3}}{4 \times 10^{-2}} = 0.06\ A = 60\ mA$.

The magnetic field which is directed into the page is decreasing, therefore the loop will work against this decrease by producing a magnetic field directed into the paper . So, by right hand rule the direction of current is clockwise .

(c) $dw = Pdt = (\frac{V^2}{R})\ dt$

$$w = \int dw = \int (\frac{V^2}{R}) dt = \int_0^1 \frac{(1.2 \times 10^{-3}\ t)^2}{4 \times 10^{-2}}\ dt$$

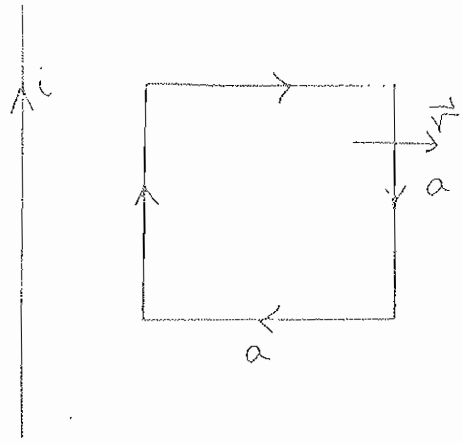
$$= (3.6 \times 10^{-5}) (\frac{t^3}{3})_0^1 = 1.2 \times 10^{-5}\ J .$$

Problem 2:

A long wire carrying a current i and a square loop of side a are in the same plane as shown

a. Find the magnetic field $\vec{B}$ at a distance r away from the wire (magnitude and direction !).

b. Calculate the magnetic flux through the square loop.



c. If the square loop is now moving with a constant velocity v to the right as shown. What is the emf around the loop? What is the direction of the induced current (draw it on the figure !)

d. If now v is made parallel to the long wire ; what are the answers to part (c) above .

Solution:

(a) Use Ampère's law. Take as a closed line integral a concentric circle of radius r .

$$\oint \vec{B}\,\vec{dl} = B(2\pi r) = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

By the right - hand - rule the direction of $\vec{B}$ is into the page .

(b) $\phi = \int \vec{B}\cdot d\vec{A} = \int_a^{2a} B.(a\ dr)$

$$= \frac{\mu_0 i\ a}{2\pi} \cdot \int_a^{2a} \frac{dr}{r} = \frac{\mu_0 \cdot i\ a}{2\pi} \ln(2).$$

(c) Let us calculate the flux at any distance r from the wire.

$$\phi = \int B(a\ dr) = \frac{\mu_0 i\ a}{2\pi} \int_r^{(r+a)} \frac{dr}{r} = \frac{\mu_0 \cdot i\ a}{2\pi} \ln\left(\frac{r+a}{r}\right)$$

$$V = \frac{-d\phi}{dt} = -\left(\frac{d\phi}{dr}\right)\cdot\left(\frac{dr}{dt}\right) = +\left(\frac{\mu_0 i\ a^2}{2\pi r(r+a)}\right)\cdot v$$

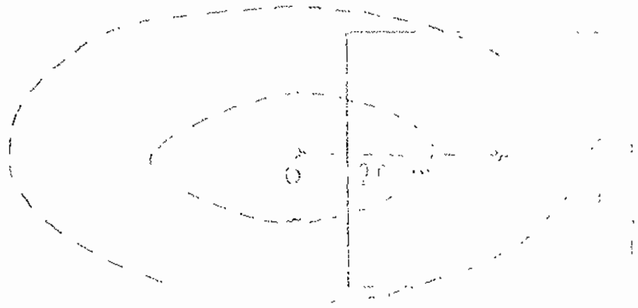
The magnetic field which is directed into the page is decreasing, therefore the loop will work against this decrease by producing a magnetic field directed into the paper. But by the right hand rule the direction of the current is clockwise to produce $\vec{B}$ directed into the page .

(d) If v is made parallel to the long wire then r doesn't change, then $\vec{B}$ does not change, then the flux does not change, therefore :

$$V = -\frac{d\phi}{dt} = 0$$, so there is no induced current.

Problem 3:

A toroid of "radius" of 20 cm (see figure) has a total of 1000 turns of wire carrying a current i. The cross section of the toroid has a radius 1 cm and is small so that $\vec{B}$ in it is approximately uniform and equal to $\vec{B}$ at the point P.



a. If $i = 5 + 2\,t^2$ Amp. where t is in seconds, find $\vec{B}$ at the point P at t = 2 sec.

b. A square loop of wire of side 5 cm and 6 turns is placed so that its plane cuts the toroid vertically, and its center is at point P (see figure). What is the magnitude and sense of the EMF induced in it at t = 2 sec.

c. If a square of side 200 cm is used instead (center still at P) what is the magnitude and sense of induced EMF now.

Solution:

(a) Let us apply Ampère's law to the circular path of integration of radius r :

$$\oint \vec{B}\, d\vec{l} = (B)(2\pi\, r) = \mu_0 i\, N$$

$$\therefore B = \frac{\mu_0\, i\, N}{2\pi\, r}$$

The radius of cross section is "small" so that for the toroid 'radius' we can take R (= 20 cm) which is the mean radius.

$$\therefore \; \frac{\mu_0 N i}{\ } = \frac{(4\pi \times 10^{-7}).(10^3)}{\ } \; (5 + 2t^2)$$

$= 10 \times 10^{-4}(5 + 2t^2)$

If $t = 2s$; $B = 0.013$ T

(b) The magnetic field is zero outside the cross sectional area of the toroid & the magnetic field is constant inside it .

$$\phi = \int \vec{B}\, d\vec{A} = BA = 10^{-3}(5 + 2t^2) \,.(\pi \times 10^{-4})$$

$$= (10^{-7} \times \pi).(5 + 2t^2)$$

$$V = \frac{-d\phi}{dt} = -(10^{-7} \times \pi).(4t) = -(4\pi \times 10^{-7})t$$

At $t = 2s$, $V = -2.51 \times 10^{-6}$ V

The emf created in the loop is such that it opposes the increase of $\vec{B}$ in the solenoid, i.e. the loop creates a $\vec{B}$ opposite to that of the solenoid . By the right hand rule we see that it creates a current in the unticlockwise direction .

(c) Now the loop contains two cross sections with equal fluxes (in magnitude) but in opposite directions.

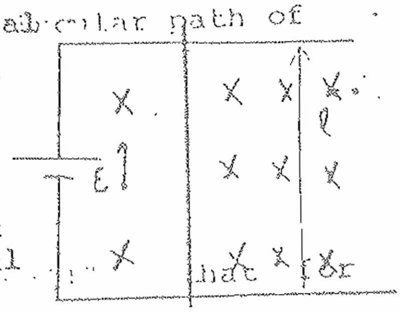
The net flux is zero

∴ The induced voltage is zero .

(no change in flux) .

Problem 4 :

A bar of resistance R = 5 and mass m = 10 g slides without friction on two parallel metal wires separated by a distance ℓ = 20 cm (see the figure below). The wires are connected to a potential difference $\mathcal{E}$ = 10 V, and an external uniform magnetic field $\vec{B}$ = 1 T is applied to the plane of the bar.



(a) What is the terminal velocity V_t reached by the bar if no external forces are applied ?

(b) Suppose a force F = 10 N is applied opposite to the direction of motion of the bar. Find the new terminal velocity in this case .

Solution:

(a) First we must understand the problem qualitatively. Initially there is a force on the bar due to i & $\vec{B}$ (directed to the right). When the bar moves to the right the area & hence the flux passing through the loop increases, thus by lenz's law there will be a current whose direction is unticlockwise which creates a $\vec{B}$ opposite to the increase of $\vec{B}$. The terminal velocity is reached when the induced voltage is equal to the initial voltage (i.e. 10 V). Or

$$V_i = Blv \qquad \text{(Blv is the induced voltage)}$$

$$10 = (1)(0.2).(v) \Rightarrow v = \frac{10}{0.2} = 50 \text{ m/s}$$

(b) In part (a) the initial force was to the right, now it is to the left, so the bar should move in a way to counterbalance this force i.e. it must induce a force to the left or create a current turning in clockwise direction. At terminal velocity there is no net force acting on the rod.

$$F = 10 = ilB \Rightarrow i = \frac{10}{lB} = \frac{10}{(0.2).(1)} = 50 \text{ A}.$$

$$V = iR = (50).(5) = 250 \text{ V}$$

Therefore the induced emf (V_i) is :

$$V_i = V - E = 250 - 10 = 240 \text{ V}.$$

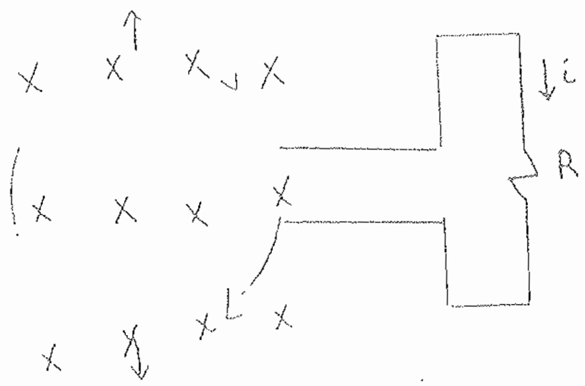
$$\text{But } V_i = Blv \Rightarrow v = \frac{V_i}{Bl} = \frac{240}{(1)(0.2)} = 1200 \text{ m/s}$$

Problem 5 :

A flexible circular loop 10 cm in diameter lies in a magnetic field 1.2 T directed into the plane of the paper . The loop is pulled at the points indicated by the arrows forming a loop of zero area in 0.2 sec.

a) Find the induced emf in the circuit.

b) What is the direction of the current in R?



Solution:

(a) The initial flux is

$$\phi = \int \vec{B}\, d\vec{A} = BA = (1.2).\left(\frac{\pi \cdot 0.01}{4}\right) = 0.0094 \text{ Wb}$$

The final flux is zero (because the area is zero)

$$\Delta\phi = \phi_F - \phi_i = 0 - 0.0094 = -0.0094 \text{ Wb.}$$

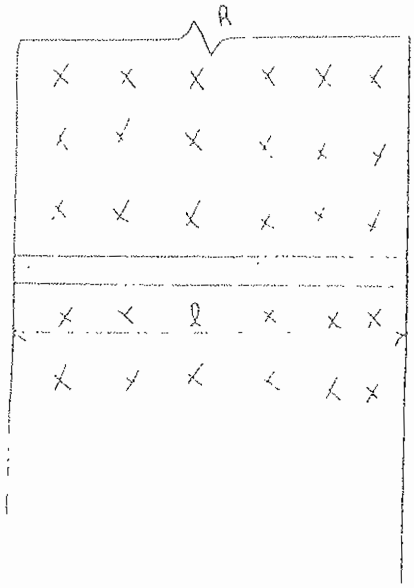
The emf induced is

$$V = -\frac{\Delta\phi}{\Delta t} = \frac{0.0094}{0.2} = 0.047 \text{ V .}$$

(b) With time the flux directed into the paper is decreased, therefore the system will act in a way to increase the flux into the paper, but the loop can create flux directed into the paper by creating current on it moving in the clockwise direction (right - hand - rule).

Problem 6 :

In the figure shown, a metallic rod of mass m and length ℓ is freely falling under the effect of gravity, but always in contact with a long frictionless metallic rail. R is the only appreciable resistance in this circuit . A magnetic induction $\overline{B}$ is applied into the plane of the page .



a- Determine the direction of the induced current. Justify your answer.

b- Determine the direction of the magnetic force in the rod . Justify your answer.

c- Find the induced current in the circuit when the rod attains its constant terminal speed .

d- Calculate this terminal speed.

Solution:

(a) When the rod is falling the flux through the loop is increasing (because the area is increasing), by Lenz's law the loop will react in a way to oppose this increase of flux, i.e. on the loop, there will be created a current moving in unticlockwise direction to create a flux directed out of the page .

(b) In the rod there is current directed to the right & the flux into the paper; by the right - hand - rule we can find the direction of force (which is magnetic) created by the current & the $\vec{B}$, this points upwards (i.e. current & the $\vec{B}$, this points upwards (i.e. opposite to the gravitational force).

(c) When the terminal velocity is reached there is no net force acting on the rod. i.e.

$$F_g = F_m \Longrightarrow mg = i.l.B \Longrightarrow i = \frac{mg}{lB}$$

(d) $$i = \frac{V}{R} = \frac{Blv}{R}$$

$$\therefore mg = \left(\frac{Blv}{R}\right)lB = \frac{B^2 l^2 v}{R}$$

$$\therefore V = \frac{Rmg}{B^2 l^2} .$$

Problem 7 :

A solenoid of radius 5 cm and length 60 cm and length 60 cm carries 1000 turns of wire. A small coil (50 turns of wire, area 2 cm^2, resistance 4 ohms) is placed at the center of the solenoid,
With its plane perpendicular to the axis of the solenoid.

(a) What will be the emf and the current induced in the coil if the current in the solenoid is changing at the rate of 4A/s.?

(b) The ratio of induced emf in the coil to the rate of change of current in the solenoid is called the mutual inductance . What is here the mutual inductance of the coil and solenoid ?

Solution:

(a) The magnetic field of the solenoid (B_s) is given as :

$$B_s = \mu_0 i n$$

$$\phi = \int \vec{B} \cdot \vec{dA} = BA = (\mu_0 n i).(A_c)$$

$$V_i = -N_c \frac{d\phi}{dt} = -(N_c)(\mu_0 n A_c) \cdot \frac{di}{dt}$$

$$= -(50).(4\pi \times 10^{-7}).\left(\frac{1000}{0.6}\right).(2 \times 10^{-4}).(4)$$

$$= 8.37 \times 10^{-5} \text{ V}$$

$$i = \frac{V_i}{R} = \frac{8.37 \times 10^{-5}}{4} = 2.1 \times 10^{-5} \text{ A}$$

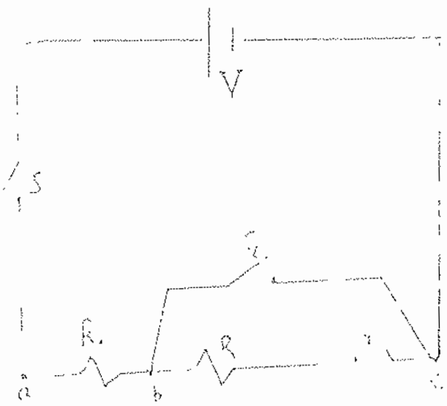
(b) $$\text{Mutual inductance} = \frac{V_i}{di/dt} = \frac{8.37 \times 10^{-5}}{4}$$

$$= 2.1 \times 10^{-5} \text{ M} .$$

[illegible]

Problem 1 :

An inductance [illegible] resistance R and self inductance L is connected in series with a resistor of resistance R_o to a constant potential V.



a) Find the expression for the potential difference V_{cb} across the inductor at any time t after switch [illegible] is closed .

b) Compute V_{cb} [illegible] t = [illegible], t = one time constant, [illegible] two time constant .

V = 20 [illegible] R_o = [illegible] , R = 150Ω , L = 5 H

c) After the current has reached its final steady value the switch S_2 is closed, short circuiting the inductor. What will [illegible] magnitude and rate change of current in S_2 [illegible] S_2 is closed .

Solution:

(a) At any [illegible] expression for the current is :

$$i = \frac{V}{R_o + R}\left(1 - e^{\frac{-(R + R_o)t}{L}}\right)$$

$$V_{CB} = V_C - V_B = -V + iR_o = -V + \frac{R_o V}{R_o + R}\left(1 - e^{\frac{-(R + R_o)t}{L}}\right)$$

(b) $V_{CB} = -20 + 5\ (1 - e^{-40t})$

At t = 0; V_{CB} = -20 + 0 = -20 V

At t = 1/40 s; V_{CB} = -20 + 5(1 - 0.36) = -16.84 V.

At t = 1/20 s; V_{CB} = -20 + 5(1 - 0.13) = -15.68 V.

(c) When the switch S_2 is closed the expression for the current in the inductor is :

$i = i_0 e^{\frac{-RT}{L}}$ where $i_o = \frac{V}{R + R_o} = \frac{20}{200} = 0.1$ A .

$\therefore i = 0.1\ 1^{-30t}$

At $t = 0.01$ s; $i = 0.074$ A.

After a long time the current through S_2 will be :

$i_T = \frac{V}{R_o} = \frac{20}{50} = 0.4$ A & the current through the inductor will be zero.

The current through S_2 at $t = 0.01$ is

$i_T - i = 0.4 - 0.074 = 0.326$ A

Rate of change of current is :(at t = 0.01s)

$\frac{di}{dt} = -\frac{R}{L} i_o e^{\frac{RT}{L}} = -(30).(0.074) = -2.22$ A/s.

Problem 2:

An inductor has an inductance of 4 H and a resistance of 4 ohm. It is connected to the terminals of a battery with an emf of 12 V and an internal resistance of 4 ohm.

a. Show that L/R has the dimensions of time.

b. Find the initial rate of increase of the current in the circuit .

c. Find the rate of increase of the current at the instant when the current is 1 A.

d. What is the instantaneous current 0.2 s after the circuit is closed ?

Solution:

(a) $\frac{L}{R} = \frac{H}{\Omega} = \frac{H}{\Omega} \cdot \left(\frac{Vs}{HA}\right) \cdot \left(\frac{\Omega\, A}{V}\right) = s$ = second.

(b) The expression for the current is :

$$i = \frac{V}{R + r}\left(1 - e^{-\frac{(R + r)t}{L}}\right)$$

$$= 1.5 \left(1 - e^{-2t}\right)$$

$$\frac{di}{dt} = (1.5).(2).\ e^{-2t} = 3e^{-2t}$$

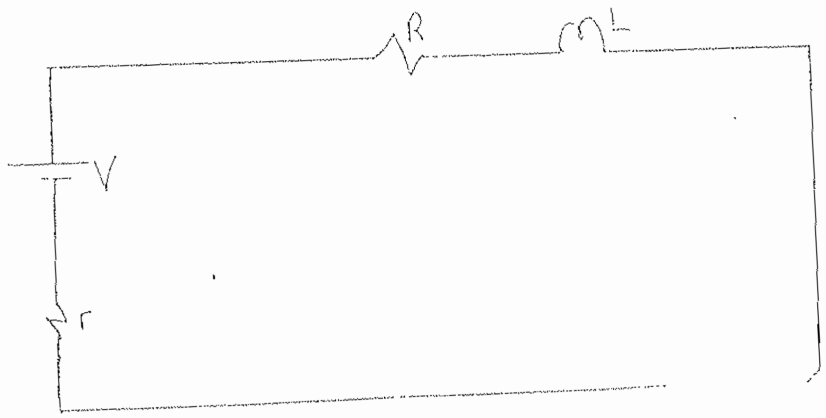
At $t = 0$; $\frac{di}{dt} = 3$ A/s.

(c) $i = 1 = 1.5(1 - e^{-2t}) \Longrightarrow 0.333 = e^{-2t} \Longrightarrow$

$1.098 = 2t \Longrightarrow t = 0.55$ s.

$di/_{dt} = 3\ e^{-2t} = 3\ e^{-(2).(0.55)} = 1$ A/s.

(d) $i = 1.5(1 - e^{-2t}) = 1.5\ (1 - e^{-0.4}) = 0.49$ A.



Problem 3 :

A long cylinder of radius a and length ℓ carries a current with a variable current density $= Ar^2$ where A is a constant .

a) Calculate the energy stored in the cylinder .

b) Deduce its inductance .

Solution:

(a) Using Ampère's law , we take the closed line integral a concentric circle of radius $0 < r < a$.

$$\oint \vec{B}\, d\vec{l} = \mu_0 i \Longrightarrow B(2\pi r) = \mu_0 i$$

Where $i = \int j\, dA = \int^r (Ar^2)(2\pi \cdot r dr)$

$= 2\pi A(\frac{r^4}{4}) = \frac{\pi \cdot Ar^4}{2}$

$\therefore B(2\pi r) = \frac{\mu_0 \pi \cdot Ar^4}{2}$

$\therefore B = \frac{\mu_0 \cdot Ar^3}{4}$

The energy density is given by :

$u = \frac{B^2}{2\mu_0} = \frac{\mu_0 A^2 r^6}{32}$

$dU = udV$

$U = \int dU = \int_0^a (\frac{\mu_0 A^2 r^6}{32}) \cdot (2\pi \cdot l r dr)$

$= \frac{\mu_0 A^2 \pi \cdot l}{16} (\frac{a^8}{8}) = \frac{\mu_0 A^2 \pi l a^8}{128}$

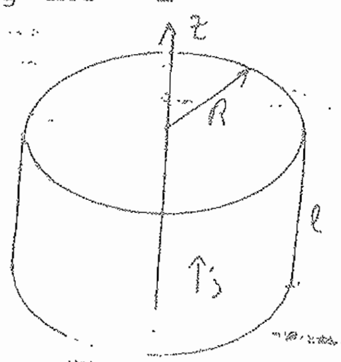
(b) $U = \frac{1}{2} L i^2 \Longrightarrow L = \frac{2U}{i^2} = (2) \cdot (\frac{\mu_0 A^2 \pi l a^8}{128}) \cdot (\frac{4}{\pi^2 A^2 a^8})$

$= \frac{\mu_0 l}{16\pi}$.

Problem 4 :

A long wire of length ℓ and radius R carries a current described by a variable current density of magnitude $j = Ar^2$ (A is a constant) and directed along the + z direction .

(a) Calculate the current in the wire .
(b) Calculate $\vec{B}$ for $r > R$.
(c) Calculate $\vec{B}$ for $r < R$.
(d) Calculate the magnetic energy stored in the magnetic field within the wire .



Solution:

(a) $I = \int j\, dA = \int_0^R (Ar^2)(2\pi \cdot r dr) = 2\pi A(\frac{R^4}{4})$

$= \frac{\pi \cdot AR^4}{2}$

(b) Using Ampère's law, we take the closed line integral

a concentric circle of radius $r > R$

$$\oint \vec{B}\, \vec{dl} = B(2\pi \cdot r) = \mu_0 i$$

Where $i = \frac{\pi \cdot AR^4}{2}$

$\therefore B = \frac{\mu_0 AR^4}{4r}$

(c) Using Ampère's law, like part (b) but $0 < r < R$

$$\oint \vec{B} \cdot \vec{dl} = B(2\pi r) = \mu_0 i$$

Where $i = \frac{\pi\ Ar^4}{2}$ (revise the previous problem)

$\therefore B = \frac{\mu_0 \cdot Ar^3}{4}$

(d) $u = \frac{B^2}{2\mu_0} = \frac{\mu_0 A^2 r^6}{32}$

$U = \int d\, U = \frac{\mu_0 A^2 \pi\, l R^8}{128}$ (revise the previous

problem).

Problem 5 :

Asolenoid is 25 cm long ,' the mean diameter is 4 cm and there are 500 turns per cm length .

a) Calculate the self inductance of the coil .

b) Calculate the energy stored , when 1 A flows in the coil .

c) Calculate the voltage across the solenoid if the circuit is broken in 0.001 s.

Solution:

(a) The magnetic field inside a solenoid is :

$B_s = \mu_0 in = (4\pi \times 10^{-7}).(i).(5 \times 10^4) = 0.063\ i\ T.$

The flux inside a solenoid is :

$\phi = \int \vec{B}\ \vec{dS} = BA = (B).(\frac{\pi d^2}{4}) = (0.063\ i).(1.25 \times 10^{-3})$

$= 7.91 \times 10^{-5}\ i$

N = 500 x 25 = 12500 turns

$$L = \frac{N\phi}{i} = \frac{(12500).(7.91 \times 10^{-5} i)}{i} = 1\ H\ .$$

(b) $U = \frac{1}{2} L\ i^2 = \frac{1}{2}(1).(1)^2 = \frac{1}{2}\ J$

(c) $V = L \frac{di}{dt} = L \frac{\Delta i}{\Delta t} = \frac{(1).(1)}{(0.001)} = 1000\ V\ .$

MAGNETIC PROPERTIES OF MATTER & ALTERNATING CURRENTS

Problem 1:

Write down an equation relating the three magnetic vectors $\vec{B}$, $\vec{H}$, and $\vec{M}$. Consider a piece of material placed in an external uniform magnetic field $\vec{B}_o$ and show the orientation and magnitudes of $\vec{B}$, $\vec{H}$, and $\vec{M}$ inside the material, when the material is

(i) Non-magnetic

(ii) Diamagnetic

(iii) Paramagnetic

What are $\vec{B}$, $\vec{H}$ & $\vec{M}$ inside a piece of permanent magnet? Show all three vectors on a diagram also.

Solution:

The equation that relates the three magnetic vectors $\vec{B}$, $\vec{H}$ & $\vec{M}$ is :

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

(i) Here the material does not have a net magnetic mome nt, so the magnetization is zero ($M = 0$) ($K_m = 1$)

$\vec{B}$ is equal to $\vec{B}_o$ in this case & substituting these values in the above equation we get $\vec{H} = \frac{B_o}{\mu_0}$

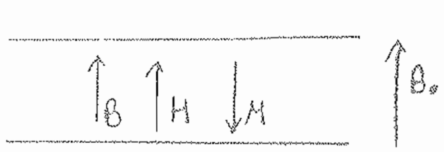
(ii). The magnitude of $\vec{B}$ is less than $\vec{B}_o$ & $\vec{B} = K_m \vec{B}_o$ (K_m is slightly less than 1)

$\vec{B} = K_m \mu_0 \vec{H} \Longrightarrow \vec{H} = \frac{B_o}{\mu_0}$

$\vec{M} = (K_m - 1)\vec{H} = (K_m - 1)\dfrac{\vec{B}_o}{\mu_o}$

But $(K_m - 1)$ is negative

indicating that $\vec{M}$ points opposite to $\vec{H}$ & $\vec{B}$



(iii) $K_m > 1$, $\vec{B} = K_m\vec{B}_o$, $\vec{H} = \dfrac{\vec{B}_o}{\mu_o}$ &

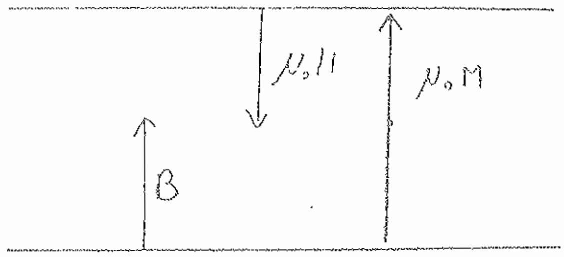
$= (K_m - 1)\dfrac{\vec{B}_o}{\mu_o}$

$(K_m - 1)$ is a positive number

$\vec{H}$, $\vec{M}$ & $\vec{B}$ have nonvanishing values inside a piece of permanent magnet; and the relation $\vec{B} = \mu_o\vec{H} + \mu_o\vec{M}$ still holds

$\oint \vec{H}d\vec{l} = i$ but there is no current in the magnet so $\vec{H}$ changes direction on the boundaries & $\vec{H}$ points opposite to magnetization inside a magnet.

(look to the figure)



We note that the magnetization is 'big' inside a magnet.

Problem 2 :

A solenoid of length 20 cm and cross sectional area 1 cm^2 has 2000 turns of wire and is filled with magnetic material of permeability $K_m = 5$.
If the current in the wire is 5 A. What are $\vec{H}$ and $\vec{B}$ inside the solenoid ? What is L ?

Solution:

Using the modified Ampère's law .

$$\int \vec{H}.\vec{dl} = i \Longrightarrow Hl = i_o N$$

$$\therefore H = i_o \frac{N}{l} = i_o n = (5).(\frac{2000}{0.2}) = 5 \times 10^4 \text{ A/m}$$

$$B_o = \mu_o H = (4\pi \times 10^{-7}).(5 \times 10^4) = 0.063 \text{ T.}$$

$$B = KmB_o = (5).(0.063) = 0.314 \text{ T}$$

$$\phi = \int \vec{B}\, \vec{dS} = BA = (0.314).(10^{-4}) = 3.14 \times 10^{-5} \text{ Wb}$$

$$L = \frac{N\phi}{i} = \frac{(2000).(3.14 \times 10^{-5})}{5} = 0.0125 \text{ H.}$$

Problem 3 :

A small iron magnet of cross-section area 1 cm^2 and length 10 cm is placed in the magnetic field of a large magnet and is attracted to the north pole of this magnet. The magnetic induction vector at the initial position of the small magnet was 1 Tesla , and at the final position (i.e. at the surface of the pole) was 2 Tesla . The density of iron is 7.9 g/cm^3 and its magnetization is 5×10^5 A/m.

a) Find the change in magnetic potential energy of the small magnet .

b) With what speed did it strike the pole ?

Solution:

(a) $M = \frac{\mu}{V}$ [illegible] $\times 10^{-4}$ [illegible] m^2

The initial potential energy is :

$U_I = - \vec{\mu} \cdot \vec{B} = -(5) \cdot (1) \cdot (\cos 0) = -5$ J

The final potential energy is :

$U_F = - \vec{\mu} \cdot \vec{B} = -(5) \cdot (2) \cdot (\cos 0)$ [illegible]

The change in magnetic [illegible]

$U = U_F - U_I = (-10) - (-5) = -5$ [illegible]

(b) Density $= \frac{m}{V}$ ⟹ $m =$ [illegible]

$= 79$ g $= 7.9 \times 10^{-2}$ Kg .

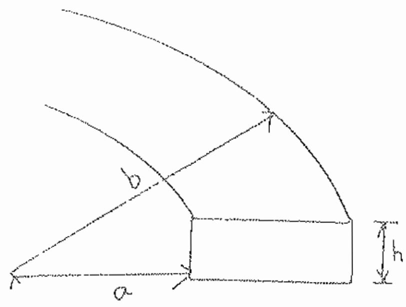
This potential energy is converted to kinetic energy.

K.E. $= \frac{1}{2} mv^2$ ⟹

$v = \left(\frac{2\ K.E.}{m}\right)^{\frac{1}{2}} = \left(\frac{2 \times 5}{7.9 \times 10^{-2}}\right)^{\frac{1}{2}} =$ [illegible].25 m/s .

Problem 4:

Derive an expression for the self inductance of a toroid of inner radius a, outer radius b and rectangular cross section of height h as shown and with a total of N turns.



How will your result change (if any) if the toroid is now filled with magnetic material of permeability $K_m > 1$.

Solution:

Using Ampère's law and taking the concentric circle of radius r ($a < r < b$) as the closed line integral :

$$\oint \vec{B} \cdot \vec{dl} = N \mu_o i \implies (B)(2\pi r) = N \mu_o i$$

$$\therefore B = \frac{\mu_o i_o N}{2\pi r}$$

$$\phi = \int \vec{B} \cdot \vec{ds} = \int_a^b \left(\frac{\mu_o iN}{2\pi r}\right) \cdot (hdr) = \frac{\mu_o iNh}{2\pi} \ln(b/a)$$

$$L = \frac{N\phi}{i} = \frac{\mu_o \cdot N^2 h}{2\pi} \ln(b/a)$$

Let the new magnetic flux be B_1

$$B_1 = K_m B \implies \phi_1 = K_m \phi$$

$$\therefore L_1 = K_m L = \frac{K_m \cdot \mu_o \cdot N^2 h \ \ln}{2\pi} (b/a)$$

If $K_m > 1$ then the inductance is increased .

Problem 5:

A steady current I is present in a long solenoid of N turns per unit length and cross sectional area A. If this solenoid is filled completely with a cylinder of magnetic material of permeability K_m [illegible] expressions for the vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$. What is the total induced dipole moment of the magnetic material?

Solution:

For a solenoid without a magnetic [illegible] magnetic field ($\vec{B}_o$) is given as

$$B_o = \mu_o IN$$

If it is filled with a magnetic [illegible]

$$B = K_m B_o = K_m \mu_o IN$$

$$B = K_m \mu_o H \Longrightarrow H = \frac{B}{K_m \mu_o} = IN.$$

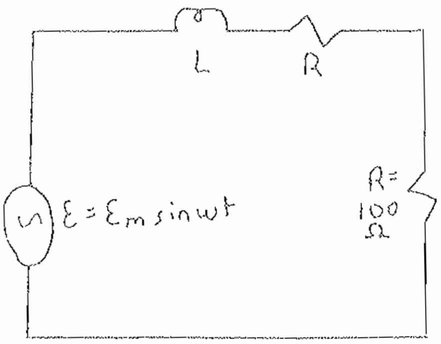
But $M = (K_m - 1) H = (K_m - 1) IN$.

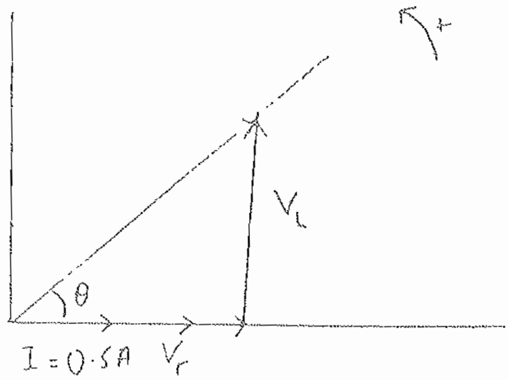
$$M = \frac{\mu}{V} \Longrightarrow \mu = M.A = (K_m [illegible] 1) INA$$

Note : This induced dipole moment is taken for a solenoid of 1 meter long. (i.e. per unit length).

Problem 6:

An inductor L which has also some resistance r is in series with a 100 Ω resistor and a 110 V, 60 Hz a.c. generator. The voltage across the resistor is found to be 50 V and the voltage across the generator is found to be leading the current by 45° . What are the values of L and r for the inductor .





Solution:

The current passing through R is equal to the current passing through the circuit.

$$i = \frac{V_R}{R} = \frac{50}{100} = 0.5\ A$$

Take the current as a referece axis .

$$Z^2 = (R + r)^2 + (wL)^2$$

Where $Z = \frac{V}{i} = \frac{110}{0.5} = 220\ \Omega$

$\therefore\ 48400 = (100 + r)^2 + (377\ L)^2$

$\tan\theta = \tan 45 = 1 = \frac{wL}{R+r} \Longrightarrow 100 + r = 377\ L$

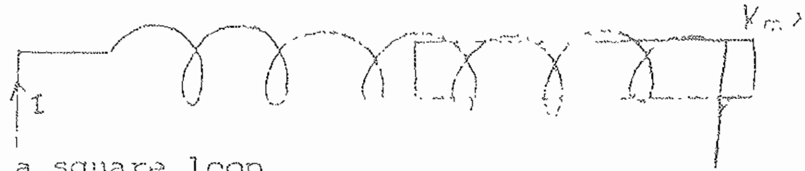
Solving for these two equations, we get .

$r = 55.56\ \Omega$ & $= 0.412$ H.

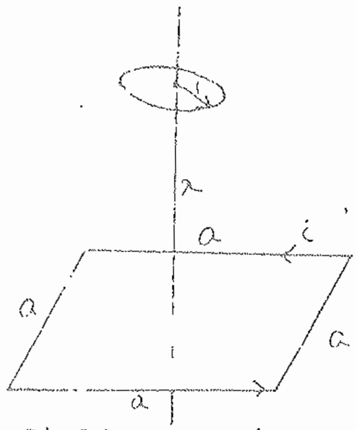
Extra Problems . (From finals)

** Consider a solenoid of current I, number of turns per unit length N_1 cross - section A , filled with a magnetic material of $K_m > 1$. Assume that the cylinder of magnetic material is inserted only half way through the solenoid & held there. Describe & explain what happens to this cylinder if it is released?

Calculate (approximately) the force that would act on the cylinder .



** This figure shows a square loop of side a and a circular loop of radius r (r < a) having the same axis. The two loops are separated by a distance X (X >> a). Hence with current $i = i_o e^{-t}$ (t = time) flowing as indicated in the square loop, the consequent magnetic field , at time t, is nearly constant throughout the plane bounded by the smaller loop .



a. Calculate the magnetic flux through the circular loop at t = 0 .

b. Calculate the induced e.m.f. in the circular loop at t = 0.

c. What is the direction of the induced current as seen by an observer at point O.

** In an RCL series circuit, the driving emf is given by $\mathcal{E} = \mathcal{E}_m \sin(wt)$ and has an internal resistance r. Take r = 200 Ω , L = 1 H, C = 5 uF, $\mathcal{E}_m$ = 300 V.

a. What is the average power P_{av} , R dissipated in the resistor R. Take R = 50 Ω , and ω = 400 rad/sec ?

b. For what value of R would P_{av}, R be maximum . Take ω = 400 rad/sec?

c. And what would this maximum power be at resonance ? What is the resonance frequency ?

** a) write curie's Law and state it in words .

b) What is the Curie temperature ?

c) Consider a cylindrical sample of a substance placed on the axis of a box magnet . Which of the following statements is correct :

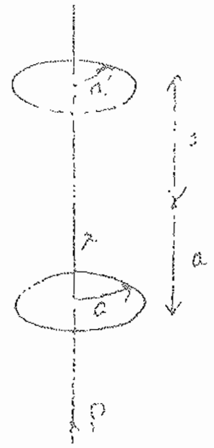
1. if the substance is paramagnetic it will be attracted by the magnet .
2. if the substance is diamagnetic it will be repelled by the magnet .
3. if the sample is itself a small magnet it may be attracted or repelled by the magnet .

d) Draw roughly the lines of induction of a bar magnet and the lines of force of an electric dipole .

** Two identical rings of radius a carrying equal and opposite charges (uniformly distributed) q and −q are placed a distance 2a apart as shown in the figure.

a) Derive the expression for the electric field $\vec{E}$ at point P. Deduce the expression of E for $x \gg a$.

b) Verify the results of part (a) by calculating the electric potential at P.



CONTENTS:
